# Decentralized Mean Orbit-Element Formation Guidance, Navigation, and Control: Part 1

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**Decentralized relative formation flight about an estimated weighted mean orbit element formation barycenter is investigated. Consensus over random directed graphs in the presence of time delays is reviewed and applied to the problem. On-orbit spacecraft formations are considered random directed graphs with time delays and agreement over formation parameters is shown. Consensus of formation state estimates and formation barycenter using a distributed formation sensor network is proven. A simulation is described, demonstrating the functionality and applicability of the approach. Conclusions and future work are discussed.**

## I. Introduction

This paper is the first portion of a two-part discussion on a Formation Flight (FF) Guidance, Navigation, and Control (GNC) solution that introduces a decentralized formation control approach. On-orbit FF GNC has received significant attention over the last decade. The need for solutions to this problem has become more immediate in recent years due to emerging mission concepts such as fractionation.[1] The fractionation approach is predicated on functionally decomposing large spacecraft into smaller, specialized spacecraft, drastically increasing the system complexity of on-board autonomous fault management systems. As the number of spacecraft in a formation increases, the burden on ground operations rises significantly. Combined with large gaps in uplink/downlink connectivity, it is infeasible to simultaneously control the entire formation from a central ground location. Further, on-orbit centralized control of the formation places the safety of the entire formation at the mercy of continued operation of a specific spacecraft. For these reasons a decentralized GNC implementation is desirable.

The on-orbit dynamic environment subjects individual spacecraft to significant oscillating, gross, and differential perturbations. Mean orbit elements effectively 'average out' oscillating disturbances[2] and are valid for all orbit regimes. This makes differential mean orbit elements particularly useful for specifying relative spacecraft geometry. Gross perturbations affect the entire formation, and can incur substantial $\delta v$ costs if they are rejected. Rather, it is preferable for long-term orbit maintenance that gross perturbations be ignored in the short term and relative formation maintenance enforced. Differential mean orbit elements relative to a weighted formation barycenter are used for a number of reasons. First, unperturbed differential elements are constants of motion for arbitrary orbit regimes (depending of course on the orbit elements chosen) and as such do not change with time, making formation 'slot' definitions easy and intuitive. Second, differential motion can be examined under the effects of perturbations (e.g., $J_2$) and partially mitigated using intelligent differential mean orbit element positions.[3–5] Third, because differential mean orbit elements change slowly under perturbations, they are ideal consensus variables.

The challenge facing on-orbit formation guidance, navigation, and control, in a decentralized fashion, is 1) for the individual spacecraft to agree on where the perturbed formation barycenter is located, 2) to know their desired relative state to achieve formation objectives, and 3) actuate to reduce relative state error while minimizing $\Delta v$.

Recent advances in cooperative control are immediately useful in addressing this challenge. The problem of spacecraft in a formation with intermittent communications agreeing on the location of the moving barycenter may be directly framed as a consensus problem over random directed graphs.[6–9] As each spacecraft is assumed to have its own on-board estimation capabilities (e.g. Extended Kalman Filters or Unscented

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Kalman Filters), the consensus problem needs to leverage provided estimation knowledge to quantify the uncertainty of the consensus barycenter (in a fashion similar to Distributed Kalman Filters[10] or Distributed Bayesian Estimation[11]). Distributed spacecraft formation flight has been previously investigated using graph theory.[12,13]

This paper intends to demonstrate formation consensus based on constituent spacecraft knowledge with two primary contributions. a) The definition of a distributed relative formation in terms of differential mean orbit elements and formation barycenter location consensus. and b) A consensus solution where each spacecraft first agrees on barycenter weights and differential mean orbit element 'slots,' and then proceeds to estimate the formation barycenter and associated uncertainty in a distributed fashion. To be clear, the contributions of this paper are not graph theory, consensus, or mean orbit element usage for formation flight, all of which are well established, rather the combined GNC approach using these techniques. Analytical predictions are verified using simulations. Conclusions and future work are discussed. The second paper (Part 2)[14] explores proper consensus weighting schemes, differential orbit element control, and formation stability.

## II. Graph Theory & Consensus

To place later results and discussion in context, a very brief overview of graph and consensus terminology is given here. Directed Graphs, Random Directed Graphs, and fundamental results on Random Graph Consensus are briefly defined and discussed.

**Definition 1.** ***Directed Graph***
*A graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ with a set of nodes $\mathcal{V}$ and a set of directed edges $\mathcal{E}(\mathcal{V})$ between the nodes in $\mathcal{V}$ is called a Directed Graph.*

**Definition 2.** ***Random Directed Graph***
*A graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ is said to be a Random Directed Graph if the probability that an edge $e_{j\to i}$ from node $j \in \mathcal{V}$ to node $i \in \mathcal{V}$, $j \neq i$, is a member of $\mathcal{E}$ with probability $p_{ij}$. This is often signified using the adjacency matrix $\boldsymbol{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ by the definition $a_{ij} = 1$ with probability $p_{ij}$, and $a_{ij} = 0$ with probability $1 - p_{ij}$.*

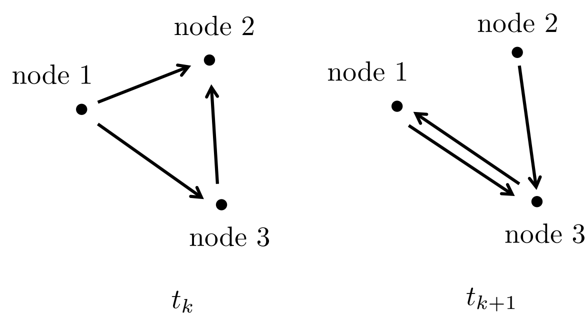


**Figure 1. Notional illustration of a random directed graph at time $t_k$ and $t_{k+1}$. Edge existence from node $j$ to node $i$ is a function of the probability $p_{ij}$.**

Broadly, consensus algorithms are concerned with independent agents represented as nodes in a graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ agreeing on the value of a consensus variable $\mathbf{x}$ (i.e. $\mathbf{x}^1(t) = \cdots = \mathbf{x}^i(t) = \cdots = \mathbf{x}^N(t)$). Consensus algorithms on the $i^{th}$ node typically operate by updating their consensus variable according to the new information obtained from the instantaneously connected $j^{th}$ node.[7,8]

$$\dot{\mathbf{x}}^i(t) = a_{ij}(t)\left(\mathbf{x}^j(t-\sigma_{ij}) - \mathbf{x}^i(t)\right) \tag{1}$$

where $\sigma_{ij}$ is the time delay of the information from node $j$ to node $i$. There exists a large body of literature that discusses the convergence of consensus problems.[6,7,13] A primary result is that, for a network of agents described by a random directed graph, if the directed interaction topology between each node $i$ and all other nodes in the graph be constructed as a tree after sufficient periods of time, then the consensus variable will lie within the convex hull of each agents initial value of the consensus variable $\mathbf{x}^i(t_0)$. This is equivalent to the union of the instantaneous topologies forming a complete graph with sufficient frequency. Further, given arbitrarily large periods of time, the network will converge on the consensus variable 'almost surely'.[6]

There are some limitations on the size of acceptable information time delays $\sigma_{ij}$. For a simple case, if $\sigma_{ij} = \sigma$, then it is necessary that $0 \leq \sigma < 2\pi/\lambda_{max}(\mathbf{L})$, where $\mathbf{L}$ is the graph Laplacian.[8,15] Hereafter, it is assumed that any time delays $\sigma_{ij} = \sigma$ satisfy this bound.

## III. Formation Definition & Results

In this section the definitions of the weighted mean orbit element barycenter, formation slots, and relative formations are given, and the resulting differential mean orbit element estimation and control problems are described. The central results of this paper - that distributed networks of sensors on spacecraft in a formation, can agree upon a common barycenter (as well as one another's states) over a communications topology represented by random directed graphs, are then proven. Finally, some discussion regarding these results is given.

Given an instantaneous set of orbit elements $\boldsymbol{œ}_k^i$ for the $i^{th}$ spacecraft in a formation $\mathcal{F}$ (notationally, $i \in \mathcal{F}$) at time $t_k$, the mean orbit elements may be directly computed using

$$\bar{\boldsymbol{œ}}_k^i = \boldsymbol{\xi}(\boldsymbol{œ}_k^i, t_k)$$

where $\boldsymbol{œ}_k^i$ are the instantaneous orbit elements of the $i^{th}$ spacecraft at time $t_k$, and $\bar{\boldsymbol{œ}}_k^i$ are the mean orbit elements of the same spacecraft at $t_k$. Mean orbit elements are the secular portion of the short and long term oscillations due to $J_2$ perturbations. It is important to mention that no specific orbit element coordinates are identified; any orbit element set will do. The definitions of the formation mean orbit element barycenter and formation slots are now given and followed by a more formal definition of a spacecraft formation.

**Definition 3.** ***Weighted Formation Barycenter***
*The formation mean orbit element barycenter $\bar{\boldsymbol{œ}}_k^b$ at time $t_k$ given spacecraft weights $w_k^i$ and reference differential mean orbit elements $\delta\bar{\boldsymbol{œ}}_{r,k}^i$ is defined as*

$$\bar{\boldsymbol{œ}}_k^b = \sum_{i=1}^{N_f} w_k^i \bar{\boldsymbol{œ}}_k^{b,i} = \sum_{i=1}^{N_f} w_k^i \left(\bar{\boldsymbol{œ}}_k^i - \delta\bar{\boldsymbol{œ}}_{r,k}^i\right) \tag{2}$$

*where the weights $w_k^i \in \mathbb{R}$ are such that $0 \leq w_k^i \leq 1$, $i = 1, \ldots, N_f$, and $w_k^1 + \cdots + w_k^{N_f} = 1$.*

In simpler terminology, the formation mean orbit element barycenter is essentially the weighted average of the individual spacecraft mean orbit elements offset by each spacecraft's differential mean orbit element. The differential mean orbit element reference offset $\delta\bar{\boldsymbol{œ}}_{r,k}^i$ may also be defined implicitly given a weighted formation barycenter $\bar{\boldsymbol{œ}}_k^b$ and instantaneous reference mean orbit element $\bar{\boldsymbol{œ}}_{r,k}^i$.

$$\delta\bar{\boldsymbol{œ}}_{r,k}^i = \bar{\boldsymbol{œ}}_{r,k}^i - \bar{\boldsymbol{œ}}_k^b \tag{3}$$

Note that $\bar{\boldsymbol{œ}}_{r,k}^i$ is the reference mean orbit elements, not the instantaneous orbit elements $\bar{\boldsymbol{œ}}_k^i$.

**Definition 4.** ***Formation Slot***
*A formation slot is defined by a spacecraft's choice of $\delta\bar{\boldsymbol{œ}}_k^i$. The only constraints placed on $\delta\bar{\boldsymbol{œ}}_k^i$ is that they be well defined on the orbit element space (e.g., differential mean eccentricity $\bar{e}_k^i$ such that $0 \leq e_k^b + \delta\bar{e}_k^i < 1$) and satisfy user-defined constraints, such as collision avoidance and other operational needs.*

Note, specific 'user-defined constraints' are introduced and discussed in the companion paper.[14] Finally, now that both the weighted formation barycenter and formation slots are defined (Definitions 3 and 4, respectively), the definition for a relative formation is now given.

**Definition 5.** ***Relative Formation***
*A spacecraft formation is said to be a relative formation when each spacecraft is aware of all other formation spacecraft (i.e. given a set of formation spacecraft $\mathcal{F}$, the size of $\mathcal{F}$, $N_f$, is known, and a specific spacecraft is associated with each $i \in \mathcal{F}$), their respective barycenter estimates $\bar{\boldsymbol{œ}}_k^{b,i}$, and barycenter weightings $w_k^i$. Further, each spacecraft must know its own differential mean orbit element formation slot $\delta\bar{\boldsymbol{œ}}_{r,k}^i$.*

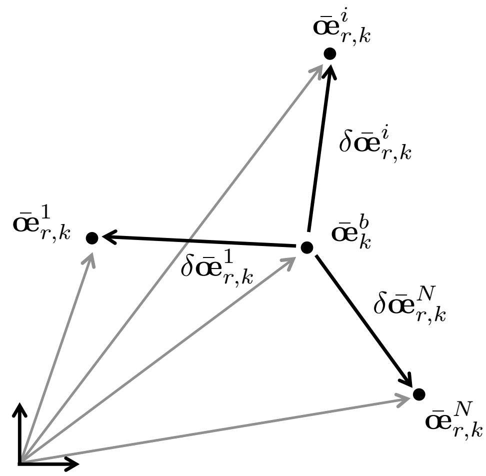


**Figure 2. Formation construction illustration. Each spacecraft maintains a relative differential orbit element offset $\delta\bar{\text{œ}}^i_k$ to the agreed upon weighted mean orbit element barycenter of the formation $\bar{\text{œ}}^b_k$.**

Note that, by definition, for any $0 < w^i_k \leq 1$, it is necessary to have $\bar{\text{œ}}^i_k$ and $\delta\bar{\text{œ}}^i_{r,k}$ to compute $\bar{\text{œ}}^{b,i}_k$, and hence $\bar{\text{œ}}^b_k$. Conversely, it is clear that if $w^i_k$, $\bar{\text{œ}}^i_k$, and $\delta\bar{\text{œ}}^i_{r,k}$ are known for all spacecraft $i = 1, \ldots, N_f$, then the formation mean orbit element barycenter $\bar{\text{œ}}^b_k$ may be computed using (2). A relative formation, weighted formation barycenter, and individual spacecraft formation slots are illustrated in Figure 2.

Importantly, given a constraint such as (3) for each slot $i = 1, \ldots, N_f$, combined with the barycenter definition (2), introduces $6(N_f - 1)$ constraints. Thus, while each individual spacecraft may be actively maneuvering to a specified relative mean orbit element position in a formation, the formation as a whole is free to drift according to collective or perturbative effects. This is a central concept in the approach outlined by this paper. If the barycenter is required to be stationary, each individual spacecraft in the formation must reject common perturbations (e.g., $J_2$, drag) in addition to differential perturbations (e.g., differential $J_2$, differential drag). Operationally, formation-wide maneuvers may periodically be required for orbit maintenance. The instantaneous value of the differential mean orbit elements is given by

$$\delta\bar{\text{œ}}^i_k = \bar{\text{œ}}^i_k - \bar{\text{œ}}^b_k \tag{4}$$

Explicitly, the instantaneous differential mean orbit element error is defined as

$$\delta\bar{\mathbf{e}}^i_k = \delta\bar{\text{œ}}^i_{r,k} - \delta\bar{\text{œ}}^i_k \tag{5}$$

Thus, the control problem described in part 2 of this effort[14] is largely concerned with intelligent ways to control the error $\delta\bar{\mathbf{e}}^i_k$ ($\delta\bar{\mathbf{e}}^i_k \to \mathbf{0}$) between the reference and instantaneous mean orbit elements for each spacecraft while still maintaining formation stability.

In operational systems it is necessary to account for sensor accuracies and knowledge as well as the overall estimation of the spacecraft formation barycenter. Suppose that some spacecraft subset $\mathcal{S}$ of the formation $\mathcal{F}$ ($\mathcal{S} \subseteq \mathcal{F}$) each have sensors capable of measuring quantities related to the individual spacecraft states $\bar{\text{œ}}^i_k$ (note that $\boldsymbol{\xi}(\cdot)$ defines a one-to-one and onto mapping between $\bar{\text{œ}}^i_k$ and $\text{œ}^i_k$). The measurement function for each spacecraft $j \in \mathcal{S} \subseteq \mathcal{F}$ sensing spacecraft(s) $p \in \mathcal{F}$ is given as

$$\mathbf{y}^j_k = \begin{bmatrix} \vdots \\ \mathbf{h}^{jp}_k(\bar{\text{œ}}^j_k, \bar{\text{œ}}^p_k, t_k) \\ \vdots \end{bmatrix} \tag{6}$$

The central results of this paper are now given in Lemma 1, Lemma 2, and Corollary 1.

**Lemma 1.** ***Formation Parameter Consensus***

*Given a formation $\mathcal{F}$ with $N_f$ spacecraft, a communication topology described by a random directed graph*

*$\mathcal{G}(\mathcal{V},\mathcal{E})$ where the $i^{th}$ spacecraft accepts all information regarding satellites $j \in \mathcal{F}$, $j \neq i$ and ignores information regarding its own consensus variable, and communication time delays not exceeding $\sigma$, the formation $\mathcal{F}$ will converge upon a consensus of individual weights $w_k^i$ and differential mean orbit element slots $\delta\bar{\boldsymbol{\text{æ}}}_{r,k}^i$ over a sufficiently large time interval $t \in [t_0, t_f]$ 'almost surely.'*

***Proof:*** *Defining the network consensus variable as*

$$\boldsymbol{x}^T = \begin{bmatrix} w_k^1 & \cdots & w_k^{N_f} & (\delta\bar{\boldsymbol{\text{æ}}}_{r,k}^1)^T & \cdots & (\delta\bar{\boldsymbol{\text{æ}}}_{r,k}^{N_f})^T \end{bmatrix}$$

*if each spacecraft $i$ ignores updates regarding its own weight $w_k^i$ and differential mean orbit element slot $\delta\bar{\boldsymbol{\text{æ}}}_{r,k}^i$ using a modified form of (1) such as*

$$\dot{\boldsymbol{x}}^i = a_{ij}\left[\boldsymbol{m}^i\right]^T\left(\boldsymbol{x}^j(t-\sigma_{ij}) - \boldsymbol{x}^i(t)\right)$$

*where the matrix $\boldsymbol{m}^i$ has zeroes where multiplication with $\dot{w}_k^j(t-\sigma_{ij}) - \dot{w}_k^i$ and $\delta\dot{\bar{\boldsymbol{\text{æ}}}}_{r,k}^j(t-\sigma_{ij}) - \delta\dot{\bar{\boldsymbol{\text{æ}}}}_{r,k}^j$ occur, then is is clear that for spacecraft $i$ (and only spacecraft $i$), $\dot{w}_k^i = 0$ and $\delta\dot{\bar{\boldsymbol{\text{æ}}}}_{r,k}^i = \boldsymbol{0}$ (effectively ensuring that weights and differential orbit elements do not have dynamics). Thus, leveraging established results for consensus (Mesbahi,[9] Tsitsiklis[6]), each spacecraft $i$ will retain its internal value of $w_k^i$ and $\delta\bar{\boldsymbol{\text{æ}}}_{r,k}^i$ while simultaneously agreeing on all other spacecraft weightings and differential mean orbit element slots.*

□

**Lemma 2.** ***Distributed Sensor Network Formation Consensus***

*Given a formation $\mathcal{F}$ with $N_f$ spacecraft, a communication topology described by a random directed graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, communication time delays not exceeding $\sigma$, a set of sensing spacecraft $\mathcal{S} \subseteq \mathcal{F}$ with measurement functions described by (6), and mutually agreed upon values $w_k^i$ and $\delta\bar{\boldsymbol{\text{æ}}}_{r,k}^i$ $(i = 1, \ldots, N_f)$, then if all of the states*

$$\tilde{\boldsymbol{\text{æ}}}_k^T = \begin{bmatrix} (\bar{\boldsymbol{\text{æ}}}_k^1)^T & \cdots & (\bar{\boldsymbol{\text{æ}}}_k^i)^T & \cdots & (\bar{\boldsymbol{\text{æ}}}_k^N)^T \end{bmatrix}$$

*are observable over a given sufficiently long time interval $t \in [t_0, t_f]$, and $\hat{\tilde{\text{æ}}}_0$ (note, the $\hat{\cdot}$ notation implies that the item $\cdot$ in question is an estimate) and $\boldsymbol{P}_0$ are chosen such that the applied nonlinear estimation algorithm is stable, the relative formation (Definition 5) will converge upon a consensus value of an estimated $\hat{\tilde{\boldsymbol{\text{æ}}}}_k$ and associated uncertainty covariance matrix $\boldsymbol{P}_k$ 'almost surely,' and as a result all spacecraft will agree on a common barycenter estimate $\hat{\bar{\boldsymbol{\text{æ}}}}_k^b$.*

***Proof:*** *From Lemma 1 it is clear that all spacecraft in the formation $\mathcal{F}$ can agree upon weightings $w_k^i$ and differential mean orbit element slots $\delta\bar{\boldsymbol{\text{æ}}}_{r,k}^i$, $i \in \mathcal{F}$. If all of the spacecraft instantaneous mean orbit elements $\bar{\boldsymbol{\text{æ}}}_k^i$ are written using $\tilde{\boldsymbol{\text{æ}}}_k$, then for each spacecraft $j$ in the set of sensing spacecraft $\mathcal{S}$ $(j \in \mathcal{S} \subseteq \mathcal{F})$ the measurement equation (6) may be written as*

$$\boldsymbol{z}_k^j = \boldsymbol{h}_k(\tilde{\boldsymbol{\text{æ}}}_k, t_k)$$

*In turn, all sensing spacecraft measurement functions may be combined such that all measurements are written as*

$$\boldsymbol{z}_k = \boldsymbol{h}_k(\bar{\boldsymbol{\text{æ}}}_k, t_k) \tag{7}$$

*Under this notation and supposing that $\tilde{\boldsymbol{\text{æ}}}_k$ is observable over the sufficiently large time interval $t \in [t_0, t_f]$, results from Extended Kalman Filters,[16] Distributed Kalman Filters (DKFs),[10] and Distributed Bayesian Estimation[11] may be directly extrapolated to the estimation and consensus of $\hat{\tilde{\boldsymbol{\text{æ}}}}_k$ and associated uncertainty covariance matrix $\boldsymbol{P}_k$. Equation (2) may then be used to compute the formation barycenter estimate $\hat{\bar{\boldsymbol{\text{æ}}}}_k^b$. It is important to recall that this result is also dependent on the stability of the chosen estimation algorithm (e.g., Extended Kalman Filter, Unscented Kalman Filter).*

□

**Remark 1.** ***Distributed Estimation Algorithm Stability***

*As mentioned in Lemma 2, for distributed formation consensus to be stable it is necessary that the distributed estimation algorithm be stable. In general the measurement functions (6) (and by extension (7)) are nonlinear, necessitating nonlinear estimation algorithms such as Extended Kalman Filters (EKFs) or Unscented Kalman Filters (UKFs). These estimation algorithms are not guaranteed to converge for arbitrary initial guesses and distributions and must accordingly be applied carefully to this problem.*

Application of Lemma 2 presumes a distributed sensor network and combined state estimation approach. A simpler, more robust though less general form, may be constructed by assuming that each individual spacecraft is capable of estimating its own mean orbit element set $\hat{\bar{\text{œ}}}^i_k$ and associated uncertainty $\mathbf{P}^i_k$. Corollary 1 distills this approach.

**Corollary 1.** ***Independent Barycenter Consensus***
*Given a formation $\mathcal{F}$ with $N_f$ spacecraft, a communication topology described by a random directed graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, communication time delays not exceeding $\sigma$, agreed upon spacecraft weightings $w^i_k$ and differential mean orbit element slots $\delta\bar{\text{œ}}^i_{r,k}$, and individual spacecraft capability to compute $\hat{\bar{\text{œ}}}^i_k$ and $\boldsymbol{P}^i_k$, then all spacecraft $i \in \mathcal{F}$ will agree on a common estimate of the formation barycenter $\hat{\bar{\text{œ}}}^b_k$ over a sufficiently large time interval $t \in [t_0, t_f]$ 'almost surely.'*
***Proof:*** *In the same manner as agreement on formation parameters is reached (see Lemma 1), the spacecraft in formation $\mathcal{F}$ can agree upon $\hat{\bar{\text{œ}}}^i_k$ and $\boldsymbol{P}^i_k$ 'almost surely' given sufficient time. After consensus is achieved the formation barycenter may be computed using 2. Note also that this result may also be derived by requiring that $\mathcal{S} \equiv \mathcal{F}$, and each measurement function may be written as $\boldsymbol{y}^i_k = \boldsymbol{h}^i_k(\bar{\text{œ}}^i_k, t_k)$.*
□

Comparing the approaches in Lemma 2 and Corollary 1 holding sensor capabilities constant, it is clear that the distributed networked sensor estimation consensus is operationally preferable because $\mathbf{P}_k$ contains cross-correlation terms, reducing the resulting uncertainty in $\hat{\bar{\text{œ}}}^b_k$. Further, if each individual spacecraft measures only its own state, no relative measurements are made. In the event of a sensor failure the corresponding mean orbit element state becomes unobservable. Contrasting this to the distributed estimation consensus approach outlined in Lemma 2, if there are sufficient sensors a single sensor failure will not eliminate the observability of the formation states. However, the approach in Lemma 2 relies on stability of the estimation algorithm over the entire formation, while that in Corollary 1 requires only that the individual spacecraft estimators be stable. Under this definition, distributed estimation methods may be applied for the formation $\mathcal{F}$ to estimate the formation orbit elements $\tilde{\text{œ}}_k$ in a distributed manner. When this approach is used, the *estimated* $\tilde{\text{œ}}_k$, $\hat{\tilde{\text{œ}}}_k$, is considered the consensus variable update law optimally mixes information from the $i^{th}$ and $j^{th}$ spacecraft (though each spacecraft maintains it's own associated uncertainty $\mathbf{P}_k$).

**Remark 2.** ***Communicated Information for an EKF***
*Each communications packet contains several pieces of information: $\boldsymbol{H}^i_k$, the measurement function Jaccobian produced by measurements, $\boldsymbol{z}^i_k$, the measurement generated by the $i^{th}$ spacecraft , $\boldsymbol{R}^i_k$, the measurement uncertainty of the $i^{th}$ spacecraft, and $\tilde{\text{œ}}^i_k$, the $i^{th}$ spacecraft's current estimate of the consensus variable $\tilde{\text{œ}}^i_k$.*

With the navigational benefits of the proposed approach described here, the following section demonstrates a successful implementation. The companion paper[14] continuous the theoretical discussion and goes in to further detail regarding Guidance and Control aspects of the proposed combined distributed mean orbit element formation flight GNC methodology.

## IV. Simulation and Results

For these examples, the classical mean orbit elements orbit elements $\bar{\text{œ}} = [\bar{a}, \bar{e}, \bar{i}, \bar{\omega}, \bar{\Omega}, \bar{M}]^T$ are used. Here, $\bar{a}$ is the mean semi-major axis, $\bar{e}$ is the mean eccentricity, $\bar{i}$ is the mean inclination, $\bar{\omega}$ is the mean argument of periapsis, $\bar{\Omega}$ is the mean ascending node, and $\bar{M}$ is the mean mean anomaly. The formation is defined with three spacecraft ($N_f = 3$). The desired differential orbit elements are given in Table 1, and the true initial mean orbit elements are given in Table 2. The individual SC weightings $w^i_k$ are chosen to be

$$w^1_k = w^2_k = w^3_k = \frac{1}{3}, \forall k$$

To emphasize the estimation and consensus results, the spacecraft in the following examples are quiescent. Maneuvering spacecraft are considered in the companion paper.[14] In the event that spacecraft in the formation are maneuvering, then the information described in Remark 2 must also include the chosen control $\mathbf{u}^i_k$. An Extended Kalman Filter (EKF) with a consensus term is used to reach barycenter consensus. For the consensus term, the consensus gain is $\epsilon = 0.1$. In the interest of simplicity, each spacecraft is assumed to have a GPS receiver with a simplified measurement function $\mathbf{z}^i_k = \mathbf{h}(\mathbf{x}^i_k) = [\mathbb{I}\ \mathbf{0}]\mathbf{x}^i_k$. The spacecraft states in the filter are expressed in Earth-Centered-Inertial (ECI) coordinates. The initial uncertainty for each

**Table 1. Desired Formation Differential Mean Orbit Elements**

| Mean Orbit Element | $\delta\bar{œ}^1_{r,k}$ | $\delta\bar{œ}^2_{r,k}$ | $\delta\bar{œ}^3_{r,k}$ |
|---|---|---|---|
| Semi-Major Axis ($\delta\bar{a}$, km) | -0.002558 | -0.002558 | -2.558e-05 |
| Eccentricity ($\delta\bar{e}$, -) | 0.0004607 | 0.0004607 | 4.628e-06 |
| Inclination ($\delta\bar{i}$, deg) | 0.0001745 | 0.0001745 | 1.745e-06 |
| Arg. of Periapsis ($\delta\bar{\omega}$, deg) | 0.001541 | -0.005085 | -0.01745 |
| Ascending Node ($\delta\bar{\Omega}$, deg) | -0.001745 | 0.001745 | 0 |
| Mean Anomaly ($\delta\bar{M}$, deg) | 0 | 0.003491 | 0.01745 |

**Table 2. True Initial Mean Orbit Elements**

| Mean Orbit Element | $\bar{œ}^1_0$ | $\bar{œ}^2_0$ | $\bar{œ}^3_0$ |
|---|---|---|---|
| Semi-Major Axis ($\bar{a}$, km) | 6879.4645 | 6879.4844 | 6879.4781 |
| Eccentricity ($\bar{e}$, -) | 0.04948277 | 0.049481426 | 0.04901866 |
| Inclination ($\bar{i}$, deg) | 0.48913929 | 0.48913941 | 0.48905386 |
| Arg. of Periapsis ($\bar{\omega}$, deg) | 0.53550703 | 0.52882969 | 0.51629866 |
| Ascending Node ($\bar{\Omega}$, deg) | 0.78361235 | 0.78709963 | 0.78535755 |
| Mean Anomaly ($\bar{M}$, deg) | 2.6077467 | 2.6112894 | 2.6254102 |

spacecraft on-board is 5m (1-$\sigma$) in position and 0.1m/s (1-$\sigma$) in velocity. As described in Lemma 2, each spacecraft simultaneously estimates the state of each spacecraft (including itself). Also, at each time step, each spacecraft computes its own estimate of the formation mean orbit element barycenter.s

Two examples are used to illustrate successful mean orbit element barycenter consensus (Lemma 2), and are summarized in Table 3. The first is a short duration simulation with a 25% chance of spacecraft receiving measurement information from one another. The second is a longer duration example that has only a 1% chance of spacecraft receiving measurement information from one another.

**Table 3. Example Descriptions**

| Example | $p_{ij}$ | Duration |
|---|---|---|
| 1 | 0.25 | 0.05 orbit periods |
| 2 | 0.01 | 2.00 orbit periods |

The first example is summarized in Figures 3 through 6. Figure 3, 4, and 5 plots the ECI state estimate error and uncertainty for spacecraft 1, 2, and 3. Because each of the spacecraft estimates of the consensus variable include ECI state estimates for itself and the other two spacecraft, each of these estimates are superimposed over one another. Because $p_{ij} = 0.25$ in this example, it can be seen that the uncertainty for the spacecraft's own state is lower than for the other spacecraft, for which only intermittent information is available. The consensus of the mean orbit element barycenter error (as measured from truth) is plotted in Figure 6. As can be seen here, the mean orbit elements are initially in disagreement and, as the simulation progresses, achieve consensus with one another and ultimately demonstrate zero-mean error with the true mean orbit element barycenter of the formation. This result is in concordance with Lemma 2.

The second example results are summarized in Figures 7 through 10. In this example, the probability of consensus information packets being received from spacecraft $i$ by spacecraft $j$ is only $p_{ij} = 0.01$. Due to this highly unreliable information transfer, the state estimate errors and associated uncertainties in Figures 7, 8, and 9 grow appreciably during gaps in communication. The difference between each spacecraft's own state estimate and that of other formation members is pronounced. The successful consensus of the mean orbit element formation barycenter between the spacecraft is plotted in Figure 10. In Figure 10, despite the highly unpredictable communications connectivity, the formation achieves zero mean consensus error after approximately 0.25 orbits, as Lemma 2 predicts. Qualitatively, the barycenter uncertainty after consensus for example 2 in Figure 10 is much larger than that shown for example 1 in Figure 6. This result conforms to intuition, as the reliability of information sharing communications is much better in example 1 than in

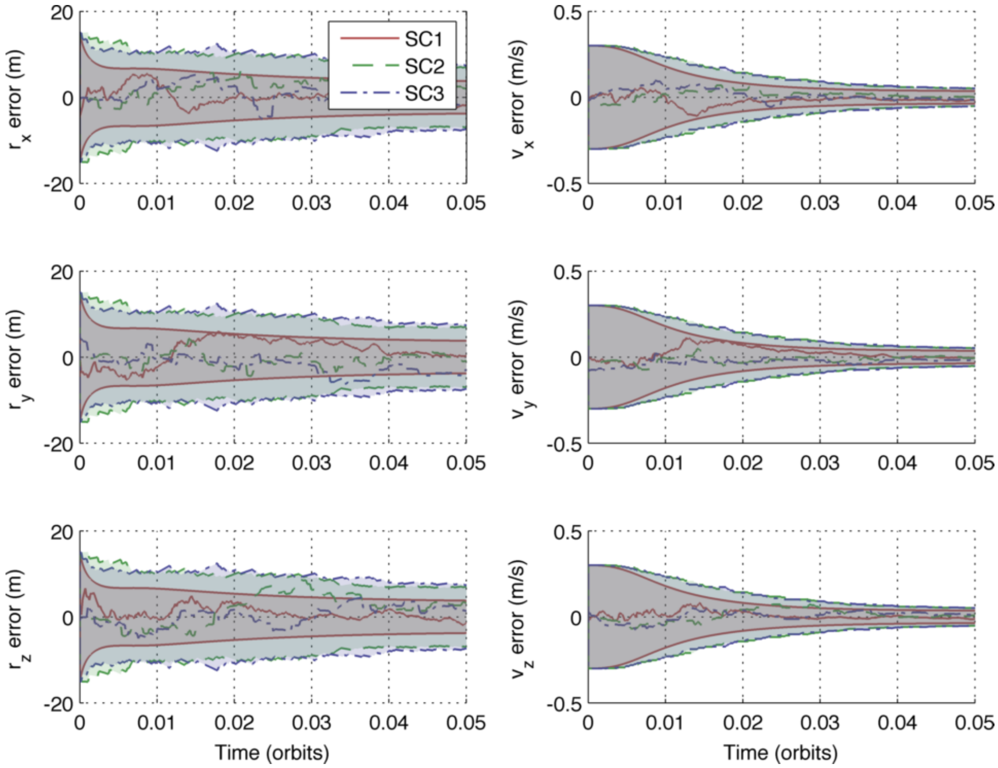


**Figure 3. Earth-Centered-Inertial EKF Estimate Error and 3-$\sigma$ Covariance for Spacecraft 1**

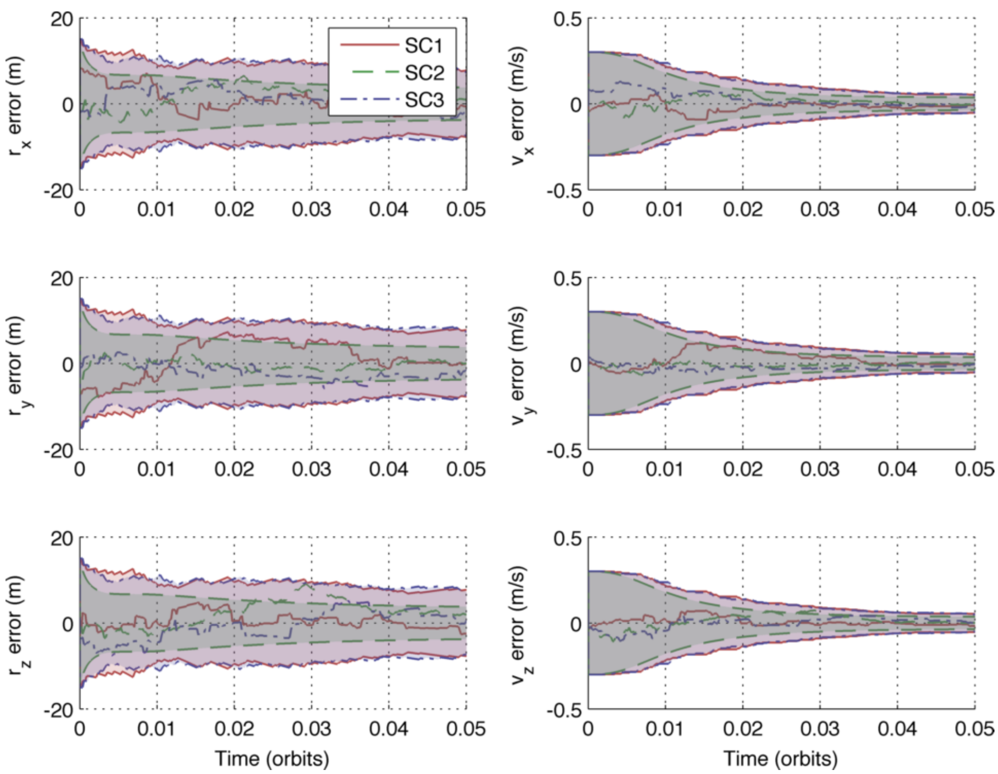


**Figure 4. Earth-Centered-Inertial EKF Estimate Error and 3-$\sigma$ Covariance for Spacecraft 2**

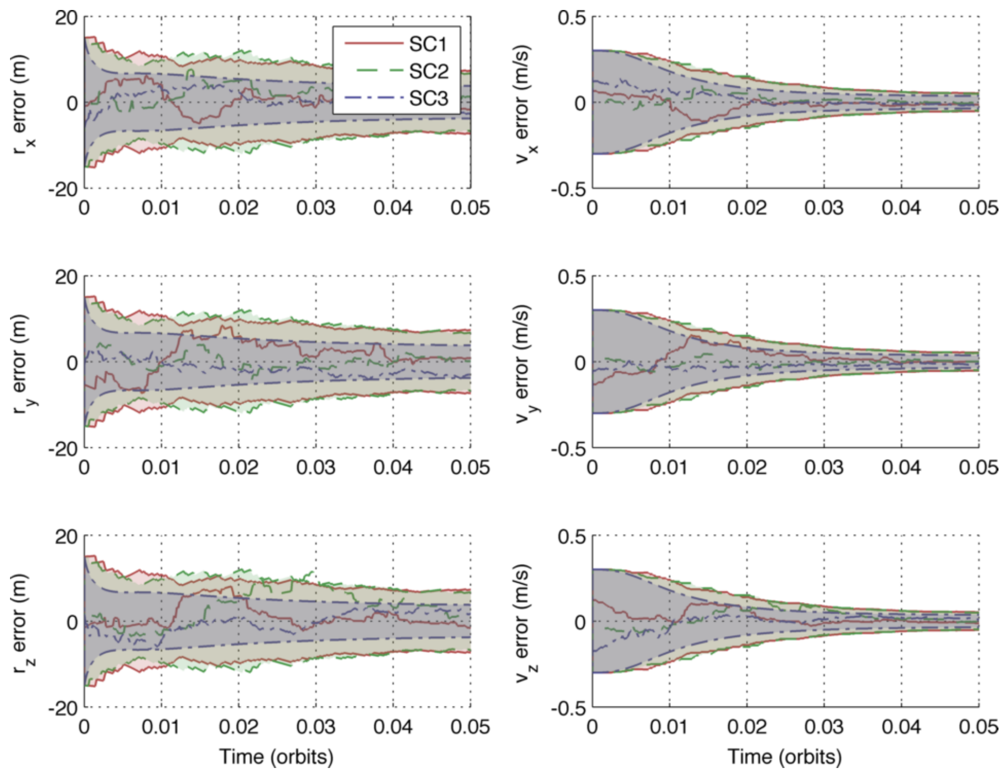


**Figure 5. Earth-Centered-Inertial EKF Estimate Error and 3-$\sigma$ Covariance for Spacecraft 3**

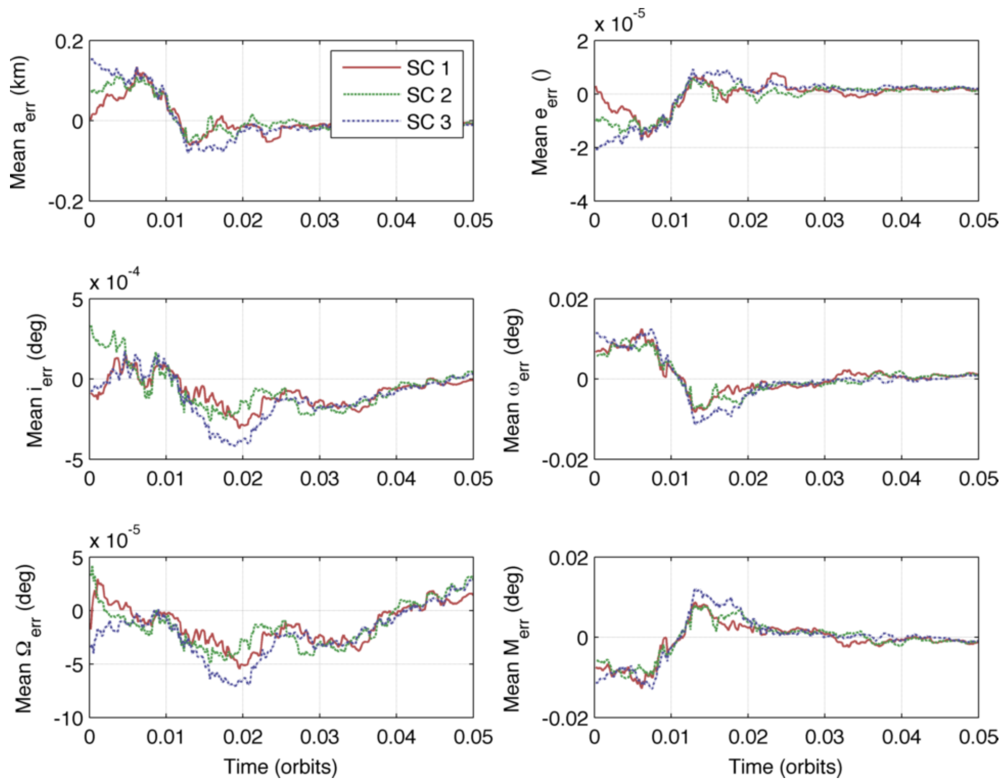


**Figure 6. Individual Spacecraft Formation Mean Orbit Element Barycenter Estimate Consensus Error**

example 2.

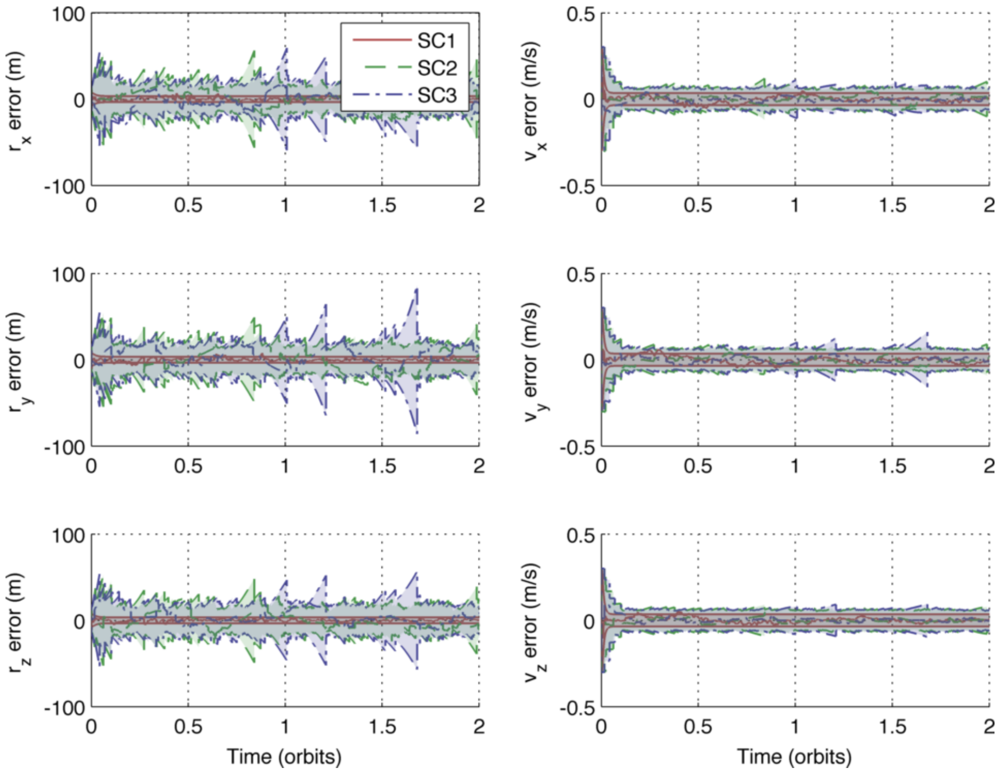


**Figure 7. Extended Earth-Centered-Inertial EKF Estimate Error and 3-$\sigma$ Covariance for Spacecraft 1**

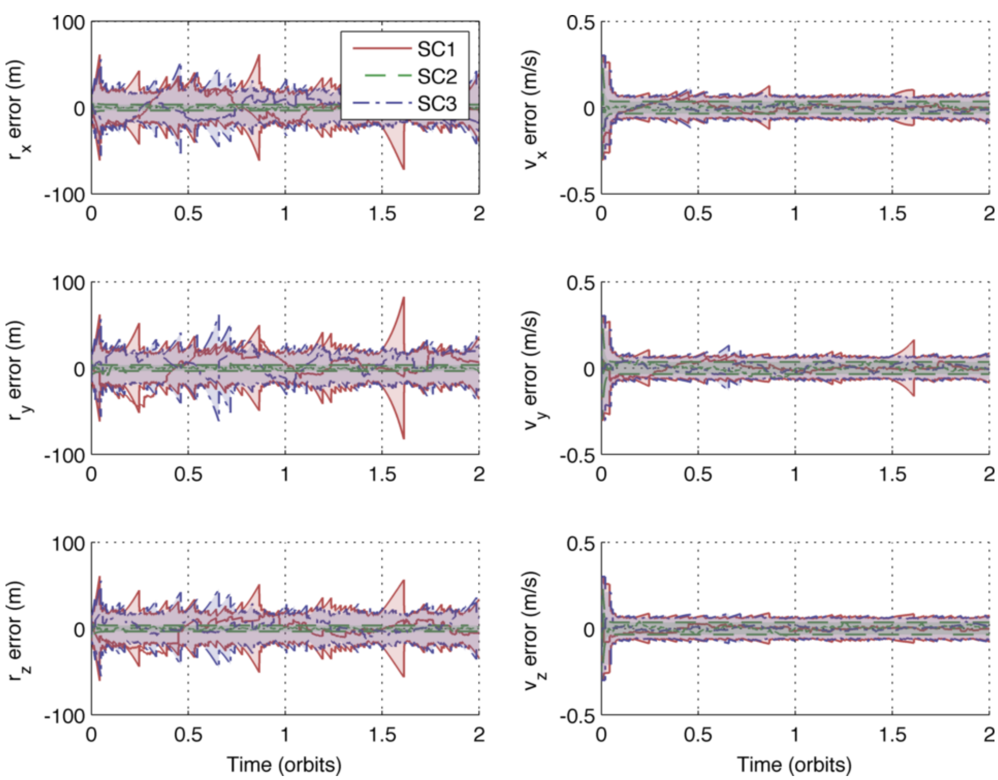


**Figure 8. Extended Earth-Centered-Inertial EKF Estimate Error and 3-$\sigma$ Covariance for Spacecraft 2**

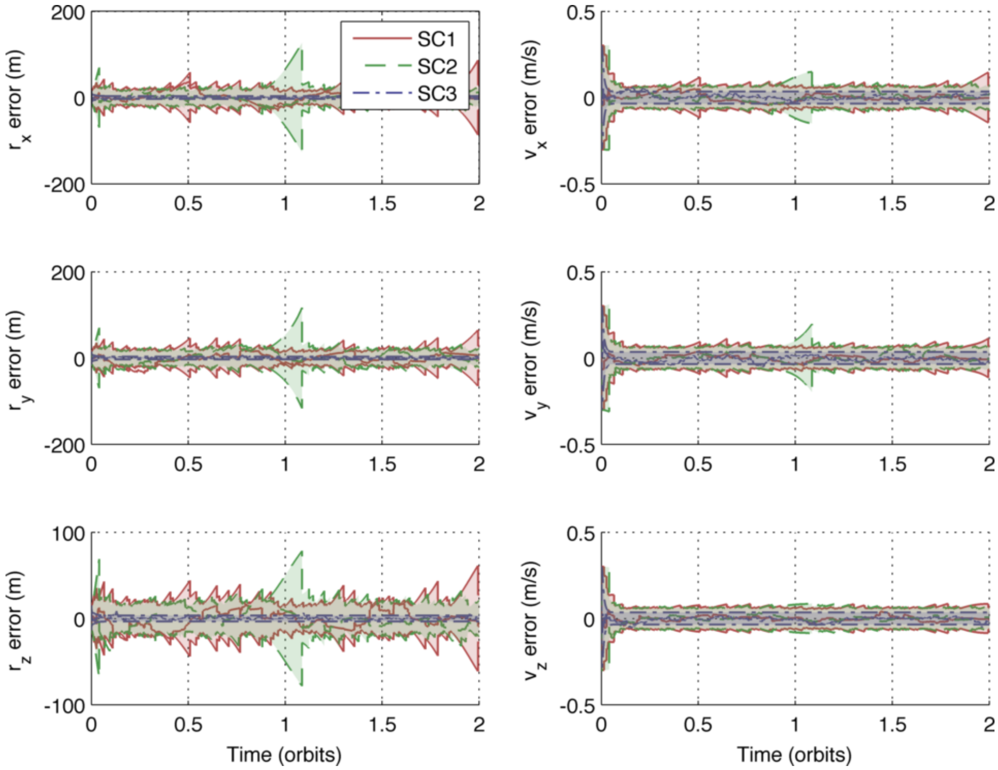


**Figure 9. Extended Earth-Centered-Inertial EKF Estimate Error and 3-$\sigma$ Covariance for Spacecraft 3**

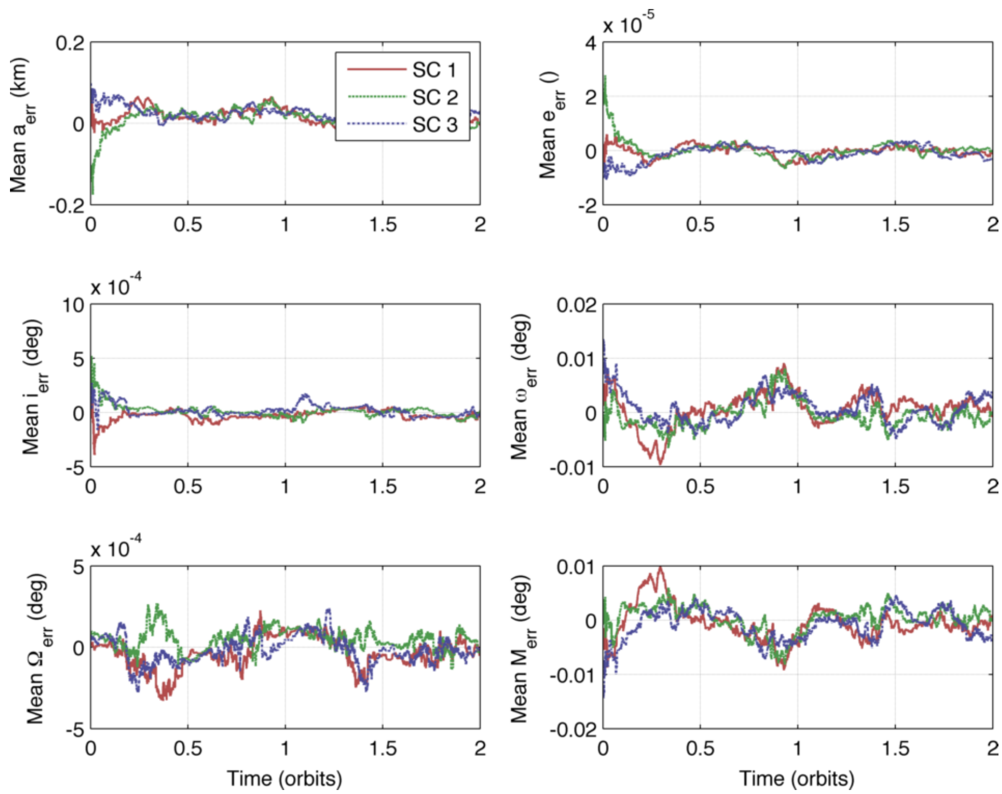


**Figure 10. Extended Individual Spacecraft Formation Mean Orbit Element Barycenter Estimate Consensus Error**

Both of these examples successfully demonstrate mean orbit element formation barycenter consensus using different directed random graph characteristics.

## V. Conclusions

A brief survey of literature for distributed systems and spacecraft formations is reviewed. Applied graph theory is briefly introduced and applied to networked groups of spacecraft in a formation. Definitions are given for weighted formation barycenters, formation slots, and relative formations. Some analytical results are presented demonstrating that, using existing theory, formations can achieve consensus on both the formation weightings and differential mean orbit elements. The guidance, navigation, and control utility of this approach to defining distributed spacecraft formations is discussed. It is then shown that by leveraging existing theoretical results in graph theory that a spacecraft formation with a distributed sensor network can achieve consensus on the spacecraft states as well as the mean orbit element formation barycenter provided that sufficient inter-spacecraft communication and state observability are present. Two examples are given that demonstrate successful mean orbit element formation barycenter consensus with different communication probabilities.

Significant opportunities for future work exist. Information-theoretic results demonstrating optimal integration of consensus variables and associated uncertainties from communicating spacecraft in the combined estimation / consensus problem would improve results shown here. Demonstration of the distributed mean orbit element combined guidance, navigation, and control problem with more interesting and non homogeneous sensor configurations (e.g., ranging, range-rate, angles-only) would also provide value. Also, the current simulation uses ECI coordinates in the filter, whereas previous work by Alfriend et al. has shown that estimation in orbit elements is demonstrably superior - future work should use orbit elements rather than ECI coordinates as the estimation state. Finally, a detailed set of operational scenario demonstrations, such as spacecraft ingress, egress, changing weightings, and various failure modes would address operational concerns related to distributed spacecraft formation flight.

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