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BINARY INTEGER LINEAR PROGRAMMING FORMULATION FOR OPTIMAL SATELLITE CONSTELLATION RECONFIGURATION

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Satellite constellations are commonly designed for fixed mission requirements. However, these systems are often subject to change in mission operations. This paper integrates the constellation transfer problem and the constellation design problem that are otherwise independent and serial in nature. Building upon the integrated model, this paper provides a solution to the following general problem statement. Suppose an existing satellite constellation system, a group of satellites with some form of shared orbital characteristics, is undertaking a reconfiguration process from its initial configuration to a final configuration due to variations in mission operations. The problem is to find the specifications of the reconfiguration process that maximizes the utility. An illustrative example is conducted to demonstrate the value of the framework.

INTRODUCTION

Satellite constellations are commonly designed for fixed mission requirements. However, these systems are often subject to change in mission operations. Potential factors that contribute to such operational variations include the change in mission coverage requirements (e.g., changing from intermittent coverage to continuous coverage), change in an area of interest (e.g., disaster monitoring, observation of new scientific events of interest, etc.), the addition of new satellites (e.g., increase in capacity[1]), and/or loss of existing satellites (e.g., due to failures,[2] or end-of-life decommissions). In such cases, it is logical for system operators to seek an option to reconfigure an existing system to fully maximize the utility of active on-orbit assets. Satellite constellation reconfiguration provides stakeholders with greater managerial flexibility to efficiently respond to both endogenous and exogenous variations in mission operations. In this paper, a satellite constellation reconfiguration is defined as a process of deliberately changing the relative positions of system satellites such that a constellation transforms from an initial orbital configuration A to a final orbital configuration B. Such a process is nontrivial and is often formulated as a large-scale brute-force design problem, which solicits theoretical development in numerous disciplines to enable a robust constellation reconfiguration framework.

Generally, the problem of reconfiguring satellite constellations is divided into two different, yet coupled, subproblems—*constellation design* and *constellation transfer*.[3,4] The former deals with an optimization problem of designing a constellation configuration given a set of static mission requirements whereas the latter deals with an optimal assignment of satellites from one configuration to another given the knowledge of both end states. Although these two problems may be approached

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independently, which have been explored by many researchers, the outcome of such a decoupled consideration may result in a suboptimal reconfiguration process as pointed out in numerous past studies.[3,5] The complexity, however, arises when the system operators wish to simultaneously optimize the design problem and the transfer problem that are otherwise independent and serial in nature. Particularly, there is an unsolved technical conundrum associated with formulating and incorporating the design features that enhance the utility of constellation reconfiguration—namely, the N-fixed formulation and regional coverage, which are discussed in the following paragraphs.

The nature of the satellite constellation design methods in the context of constellation reconfiguration dictates that the design of a reconfigured system must be constrained to the fixed number of satellites from the precedent stage (assuming no launch of new satellites nor loss of any). We refer to such a design optimization approach as the N-fixed formulation of the design problem (here, N indicates the number of satellites in a system). On the other hand, the classical methods focus on the design of optimal configurations without constraining the number of satellites where the goal is to strictly satisfy certain coverage requirements, which is beneficial when designing an initial configuration for known static mission requirements (we call this N-flexible methods).

Traditionally, satellite constellation reconfiguration problems have been focused on the staged deployment problems (e.g., increase in capacity[6,7]) and the loss reduction problems.[2] However, there is another attractive, perhaps one of the most significant but scarcely studied, design philosophy that is known to encompass the idea of constellation reconfiguration: regional coverage.[3] A satellite constellation covering a portion of the Earth can be reconfigured into a new configuration such that it provides coverage over other areas of interest. For example, when a constellation wishes to conduct a single-fold continuous coverage remote sensing task over a certain ground target or when system stakeholders wish to change the area of communications service, the constellation would need to be reconfigured into a new configuration to meet the new mission requirement. The concept of regional coverage justifies the rationale for satellite constellation reconfigurations and boosts operational flexibility under mission variations and uncertainties.

The framework introduced herein incorporates the aforementioned design features of the constellation design problem in the context of satellite constellation reconfiguration based on the APC decomposition modeling of satellite constellation architecture,[8,9] which makes the use of the repeating ground track orbits and the common ground track constellation. The main contribution of this paper lies in the development of an integrated model that enables a coupled consideration of design and transfer problems that are otherwise independent and serial in nature. The formulation of the optimization problem is of binary integer linear programming (BILP) form, which enables the use of global optimization techniques.

This paper provides a solution to the following general problem statement. Suppose an existing satellite constellation system is undertaking a reconfiguration process from its initial configuration A to a final configuration B due to variations in mission operations. The problem is to find the specifications of the reconfiguration process that maximizes the utility. Here, the specifications of the reconfiguration process refer to both the design and transport factors; the utility refers to the coverage performance and the ΔV cost incurred due to reconfiguration. The goal of this paper is to construct a framework that will allow system operators to efficiently meet the emerging operational needs. An illustrative example is conducted to demonstrate the value of the framework.

The remainder of this article is organized as follows. The second section overviews the key literature relevant to this study. The third section overviews the definitions and assumptions made in this paper. The fourth section introduces the modeling of the reconfiguration process and pertinent figures of merit. The fifth section then formulates various optimization problems based on the modeling. The sixth section demonstrates the value of the framework through a case study. The last section provides conclusions of this paper and discusses the potential extensions of this work.

LITERATURE REVIEW

The satellite constellation reconfiguration problem is an active field of space systems operations research. This section reviews the key literature relevant to this study.

Several satellite constellation patterns that finite the design space have been proposed. The most notable classical methods include the Streets-of-Coverage (SoC)[10,11] and Walker patterns.[12–14] The SoC and Walker patterns approach the constellation coverage problem from a geometric perspective and hence lead to easy-to-use global coverage constellations patterns that are symmetric in nature. Flower constellation set theory is one of the latest development in the modern constellation design theories,[15,16] which has shown to be a general framework that encompasses Walker patterns.[17] The Flower constellation leverages the concept of restricting all satellites to follow a closed 3D trajectory; it has shown to be beneficial for regional coverage purposes, which is one of the design features this paper is seeking after. Note that, despite its attractiveness, the Flower constellation, per se, is only a mathematical set theory that describes a constellation pattern, not an optimization method that designs optimal configurations.

The transfer problem, also known as the transportation problem, of the satellite reconfiguration is a relatively well-established field of study. It consists of two tasks: identifying optimal orbital transfer between initial and final orbits and the assignment of satellites. The optimal orbital transfer problem has been studied by numerous researchers, mainly exploring non-Hohmann orbital transfers.[4,18] For the assignment, de Weck et al. applied the balanced assignment problem formulation to the satellite constellation reconfiguration and solved it using the auction algorithm.[19] The nature of the assignment problem requires the given knowledge of both the initial and final configurations such that the flows between the orbital slots (i.e., the nodes) of bipartite sets be the design variables; the objective function of the assignment problem is to minimize the total cost of the flows (in their case, the cost matrix was hard-coded based on Hohmann transfer scheme).

Several satellite constellation reconfiguration studies have been proposed encompassing both the constellation design and the constellation transfer. Ferringer et al. explored a case when one or more satellites experience failures and formulated a framework that can be used to minimize the loss.[2] In his study, the state of the terminal configuration is undetermined and was found via multi-objective numerical optimization (e.g., propellant usage, coverage performance, etc.). The mathematical formulations exhibited nonlinear behaviors and therefore the ϵ-NSGA-II algorithm is used to approximate the Pareto front.[20] Several studies have explored the concept of a reconfigurable constellation (ReCon). Paek constructed an optimization framework based on the concept of a satellite constellation switching between two operational modes—global observation mode and regional observation mode—for geo-spatially adaptive Earth observation missions.[21] Legge extended this concept to optimize the overall ReCon architecture by concurrently considering constellation pattern design, satellite design, and operations design; in his approach, the ϵ-NSGA-II algorithm is also used.[5] While these efforts demonstrated the value of the concurrent optimization, their mathematical formulations were nonlinear and in the form of nested-optimization, which constrained their choice of optimization solvers to be metaheuristics algorithms.

This paper deals with the integration of constellation design and constellation transfer optimization problems. The uniqueness of this study lies in the consideration of design aspects that reinforce the argument of satellite constellation reconfigurations: the regional coverage and the N-fixed formulation. Particularly, the regional coverage feature unveils the hidden design space by enabling asymmetric constellation patterns. The constructed framework is general in the sense that it can be used to respond to various combinations of variations in mission operations.

PRELIMINARIES

This section introduces the definitions of key terms used in this paper and the assumptions that lay the foundation for the modeling of a general reconfiguration process. This article adopts and extends prior works[8,9] on the design of satellite constellations for regional coverage based on the circular convolution formulation.

Regional Coverage Satellite Constellation

Two assumptions are made about the constellation pattern based on the need for satellite constellation reconfiguration considering regional coverage: (1) the repeating ground track orbits and (2) the common ground track constellation. This subsection briefly discusses each item.

Repeating Ground Track Orbit A ground track is the trace of satellite's sub-satellite points on the surface of a planetary body. A repeating ground track (RGT) orbit mandates that if a satellite makes N_{P} number of revolutions in one period of repetition T_{r}, which is defined by the multiple of an integer N_{D} and the nodal period of Greenwich T_{G}, then the ground track of that satellite repeats exactly and periodically. Expressing this condition:

$$T_{\mathrm{r}} = N_{\mathrm{P}} T_{\mathrm{S}} = N_{\mathrm{D}} T_{\mathrm{G}} \tag{1}$$

where N_{P} and N_{D} are positive integers and T_{S} is the nodal period of a satellite due to both nominal motion and perturbations.

The practicality of an RGT orbit is greatly acknowledged in the field of satellite constellation design and optimization. The received wisdom is that an RGT orbit provides better coverage performance over a locally-bounded target than a non-RGT orbit.[22] This is because the ground track is fixed relative to the target. To find the value of a semi-major axis a that forms an RGT orbit under the Earth oblateness effect (e.g., J_2 perturbation effect), one would need to perform an iterative numerical method, such as the Newton-Raphson method, with given orbital parameters N_{P}, N_{D}, eccentricity e, and inclination i (for more information, refer to Reference 23).

Common Ground Track Constellation A common ground track constellation enforces all system satellites to follow a same 3D trajectory in a rotating frame, which is equivalent to having a common ground track when the trajectory is projected on to the surface of a planetary body. Based on the rationale for persistent regional coverage, we only consider circular orbits or elliptic orbits with critical inclinations ($i \in \{63.4°, 116.6°\}$). This is since non-critically-inclination elliptic orbits incur heavy orbital maintenance costs to maintain its apogee under perturbation effects. Note that, we consider the Earth-Centered Earth Fixed frame as the rotating frame of interest.

All system satellites in a common RGT constellation share identical semi-major axis a, eccentricity e, inclination i, and argument of perigee ω but independently hold right ascension of ascending node (RAAN) Ω and mean anomaly M pairs that satisfy the following relation:

$$N_{\mathrm{P}}\Omega_k + N_{\mathrm{D}}M_k = \text{constant mod } (2\pi) \tag{2}$$

The APC Decomposition: Coupling Configuration and Coverage Performance

The two assumptions made in the prior subsection together constitute the *cyclic property* of a common RGT constellation. The APC decomposition*, a representation of a common RGT constellation architecture via three finite-length sequences—a seed satellite access profile $\boldsymbol{v}_{0,j}$, constellation pattern vector $\boldsymbol{x}$, and coverage timeline $\boldsymbol{b}_j$, describes not only the physical configuration but also the coverage performance of the system, which is the time-dependent coverage figure of merit for satellite constellations. This section briefly introduces the APC decomposition and its pertinent definitions.

Definitions Following definitions are the core elements that describe the APC decomposition:

- Seed satellite access profile $\boldsymbol{v}_{0,j} \in \mathbb{Z}_2^L$ stores binary information whether a seed satellite—a hypothetical reference satellite that stores orbital information that is inherent to actual system satellites—has access to (or is visible from) a designated target point of interest j at each discrete-time instant n.
- Constellation pattern vector $\boldsymbol{x} \in \mathbb{Z}_2^L$ stores binary information about system satellites' relative temporal spacing with respect to the seed satellite at each discrete-time instant n.
- Coverage timeline $\boldsymbol{b}_j \in \mathbb{Z}_{\geq 0}^L$ stores information about satellite diversity (i.e., a number of satellites in view) at each discrete-time instant n. For example, if $b_j[100] = 2$, then there are two satellites that are visible from or have access to a target point j at $n = 100$.

Here, $j \in \mathcal{J}$ is the index of a target point, $\mathcal{J}$ is a set of target points, and $n \in \{0, ..., L-1\}$ refers to the discrete-time instant. A superscript z is added appropriately to each definition to specify the definition's relation with the corresponding zth subconstellation; a subconstellation is defined as a subgroup within a constellation system (a system being the system-of-subconstellations). Note that, following the convention from the digital signal processing community, this article utilizes a zero-based numbering: the index of a vector of length L ranges $[0, L-1]$.

The Circular Convolution Phenomenon A coverage timeline $\boldsymbol{b}_j$ is the result of circularly convolving a seed satellite access profile $\boldsymbol{v}_{0,j}$ with a satellite constellation pattern vector $\boldsymbol{x}$. Expressing this relation mathematically:

$$\boxed{\begin{aligned} b_j[n] = x[n] \circledast v_{0,j}[n] &= \sum_{m=0}^{L-1} x[m] v_{0,j}[(n-m) \bmod L] \\ = v_{0,j}[n] \circledast x[n] & \quad \text{(commutative property)} \end{aligned}} \tag{3}$$

The proof of this relation is referred to Reference 9. Eq. (3) can be shown in terms of a circulant matrix:

$$\boldsymbol{b}_j = \boldsymbol{V}_{0,j}\boldsymbol{x} \tag{4}$$

where $\boldsymbol{V}_{0,j} \in \mathbb{Z}_2^{L \times L}$ is a circulant matrix constructed by augmenting each column with circularly-shifted seed satellite access profiles $\boldsymbol{v}_{0,j}$.

*It is named after the acronyms of seed satellite access profile, constellation pattern vector, and coverage timeline

Using Eq. (4), one can formulate a binary integer linear program to find the N-minimizing constellation pattern $\boldsymbol{x}^*$ that strictly satisfy the given coverage requirement; this is introduced in Eq. (19). The optimization problem, hence, is in the form N-flexible formulation; the number of satellites is allowed to freely vary during the optimization process. The APC decomposition is demonstrated in the following sub-subsection.

Illustration of the APC decomposition To demonstrate the APC decomposition modeling, consider an arbitrarily defined two-satellite constellation system with a seed satellite orbital elements vector $\text{œ}_0 = [a, e, i, \omega, \Omega, M]^T = [9064.7\,\text{km}, 0, 70°, 0°, 20°, 0°]^{T*}$, which corresponds to the RGT ratio of $\tau = N_\text{P}/N_\text{D} = 10/1$ (i.e., a satellite makes 10 revolutions during 1 nodal day). A single target of interest $\mathcal{J} = \{(\phi = 60°\text{N}, \lambda = 30°\text{E})\}$ is assumed to require a minimum elevation angle threshold $\varepsilon_\text{min} = 10°$ to be visible from the on-orbit satellites, which is determined a priori based on mission requirements. (ϕ is the latitude and λ is the longitude of a target point.) Assume an arbitrarily defined satellite constellation pattern vector $\boldsymbol{x}$:

$$x[n] = \begin{cases} 1, & \text{for } n = 0, 360 \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

The constellation pattern vector in Eq. (5) is equivalent to having two satellites each with its own orbital elements vector; note that all satellites have identical a, e, i, and ω but each satellite holds an independent pair of Ω and M:

$$\text{œ}_1 = [9064.7\,\text{km}, 0, 70°, 0°, 20°, 0°]^T \tag{6a}$$

$$\text{œ}_2 = [9064.7\,\text{km}, 0, 70°, 0°, 200°, 0°]^T \tag{6b}$$

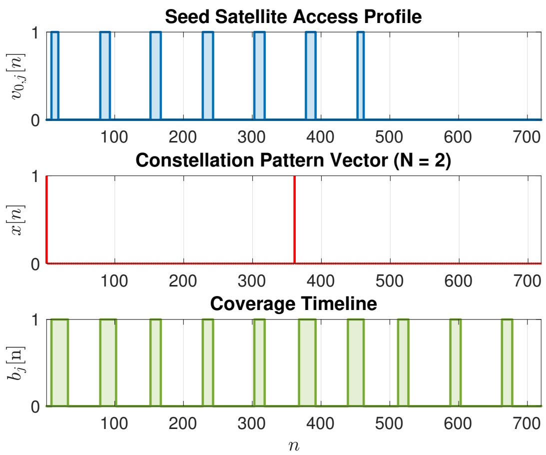
The visualization of the example system is shown in Figure 1. Each satellite exhibits identical but circularly-shifted access profiles as shown in the top part of Figure 1a. The resulting coverage timeline for this two-satellite system is shown in the bottom part of Figure 1a. The corresponding illustration of such a system in the Earth-centered inertial (ECI) frame is shown in Figure 1b, which can be shown directly from plotting the actual orbits of each satellite from specified orbital elements vectors in Eqs. (6). For this specific demonstration, the length of vectors is chosen, $L = 720$ for one period of repetition, such that it corresponds to a time-step of approximately $120\,\text{s}$. Note that all values in Eqs. (6) are referenced to the epoch 15 Feb 2017 12:00:00 UTC.

The main advantage of utilizing the APC decomposition is that it enables a linear expression of satellite constellation design space, i.e., a single $x[n]$ element is a single point in the (Ω, M)-space via coupling the time and space. Additionally, it couples each satellite constellation configuration with its corresponding coverage timeline, which can be computed analytically as shown in Eq. (3). Such a representation allows any common RGT satellite constellation configuration $\mathcal{G}$ to be expressed as a set of seed satellite orbital elements vector œ_0 and constellation pattern vector $\boldsymbol{x}$:

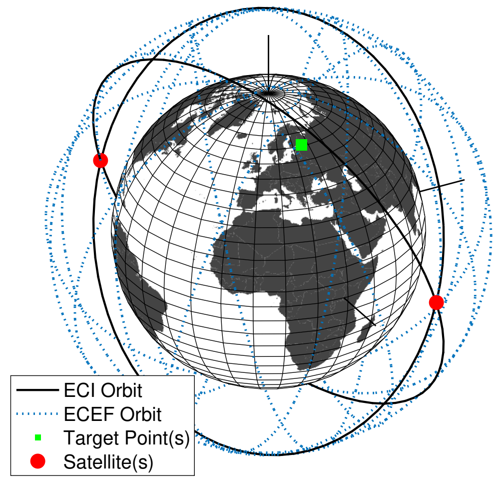
$$\boxed{\mathcal{G} = \left\{ \text{œ}_0^{(1)}, \ldots, \text{œ}_0^{(|\mathcal{Z}|)}, \boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(|\mathcal{Z}|)} \right\}} \tag{7}$$

where z is the index of a subconstellation and $|\mathcal{Z}|$ is the cardinality of a set of system sub-constellations $\mathcal{Z}$.

*Note that the mean anomaly of a seed satellite can be set to zero without loss of generality as shown in Eq. (2)



(a) The APC decomposition



(b) Corresponding configuration representation in 3D

Figure 1: The APC decomposition and its equivalent constellation representation in 3D space

The APC decomposition model is an instance of the N-flexible method class, in which the number of satellites N is free to vary. While this can be directly used to design an initial configuration, it has a limitation when we start to consider the reconfiguration problem, which is in the form of a N-fixed formulation. Therefore, it is imperative that we need a new approach. As will be discussed later in this paper, the new approach is introduced exploiting the linear property granted by the APC formulation and the linear assignment problem, which is the crux of the present study. Figure 2 visualizes the scope of this paper. The scope of this article is the reconfiguration process such that the prior works deal with the design of satellite constellations for optimal coverage; a combination of which provides comprehensive and useful tools for system designers and operators.

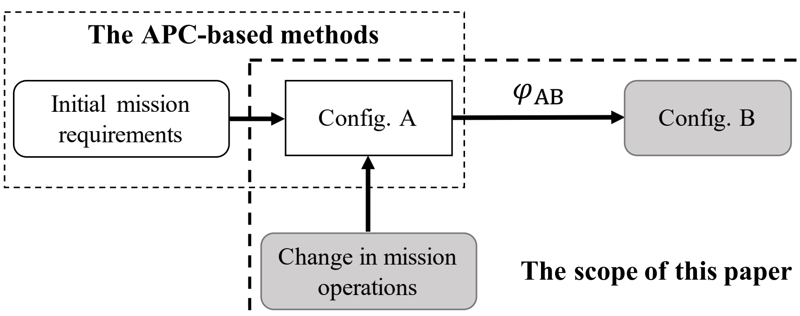


Figure 2: The scope of this paper and its alignment with respect to prior works

MODELING

This section discusses the modeling of the reconfiguration process and pertinent figures of merit that can be used to measure the optimality of satellite constellation reconfigurations.

Modeling the Reconfiguration Process

Suppose an arbitrary single-subconstellation common RGT satellite constellation configuration, $\mathcal{G}_\text{A} = \{\boldsymbol{\text{œ}}_{0,\text{A}}, \boldsymbol{x}_\text{A}\}$, undertakes a reconfiguration process due to one or more following operational variations:

1. Change in the mission coverage requirement, $\boldsymbol{r}_\text{A} \rightarrow \boldsymbol{r}_\text{B}$
   - Example coverage types: continuous coverage (single-fold, double-fold, etc.), intermittent (discontinuous) coverage, time-dependent coverage, etc.
   - Change in the minimum elevation angle threshold, $\varepsilon_{\text{min,A}} \rightarrow \varepsilon_{\text{min,B}}$
2. Change in the area of interest, $\mathcal{J}_\text{A} \rightarrow \mathcal{J}_\text{B}$
3. Change in the number of satellites, $N_\text{A} \rightarrow N_\text{B}$
   - Addition of new satellites (e.g., capacity expansion)
   - Removal of existing satellites (e.g., loss, decommission)

that necessitates the change in one or both of the following system constituent variables:

1. Change in the seed satellite orbital elements vector, $\boldsymbol{\text{œ}}_{0,\text{A}} \rightarrow \boldsymbol{\text{œ}}_{0,\text{B}}$
2. Change in the constellation pattern vector, $\boldsymbol{x}_\text{A} \rightarrow \boldsymbol{x}_\text{B}$

which results in the new coverage timeline $\boldsymbol{b}_\text{B}$.

A reconfiguration process φ_AB is defined as the state-to-state mapping from an initial satellite constellation configuration A to a final configuration B given a combination of mission operational variations. Generalizing the above steps to a multiple-subconstellation system as defined in Eq. (7),

$$\boxed{\varphi_\text{AB} : \mathcal{G}_\text{A} \rightarrow \mathcal{G}_\text{B}} \tag{8}$$

One can view a reconfiguration process φ_AB as the deliberate change in the common orbital characteristics and/or the constellation pattern of the system in response to given operational decisions or perturbations. The coverage timeline $\boldsymbol{b}_\text{B}$ of the final constellation configuration B is the output of such a mapping. Note that the reconfiguration process is a "mapping" that describes which satellite in configuration A moves to which orbital slot in configuration B. Hence, specifications of a reconfiguration process involves both the design of configuration B as well as the assignment of satellites.

We can model the addition or the removal of satellites during a reconfiguration process as:

$$N_\text{B} = N_\text{A} + \alpha = \sum_{n=0}^{L-1} x_\text{A}[n] + \alpha \tag{9}$$

where $\alpha \in \mathbb{Z}$ represents a change in the number of satellites for the subsequent stage. If $\alpha > 0$, α number of satellite(s) are being added to the subsequent stage. We make an assumption that α number of satellites are initially positioned at $n = 0$, where satellites are deployed from a launcher. This is equivalent to having $x_\text{A}[0] = \alpha$. On the other hand, the loss or removal of α number of satellite(s) is assumed when $\alpha < 0$. In this case, the specific referral of which satellites from the initial configuration are removed is required (e.g., removal of $k = 3$ satellite). Likewise, if $\alpha = 0$, then no addition or removal of satellites from the preceding stage is assumed.

Quantifying the Performance and the Cost of the Reconfiguration Process

The prior subsection introduced the model that describes a reconfiguration process from an initial configuration A to a final configuration B (e.g., inputs and outputs). To assess the quality of the reconfiguration process, we introduce two figures of merit: the coverage performance and the cost incurred due to the state transition. The following two sub-subsections discuss each item in detail.

Coverage Figure of Merit The N-fixed formulation dictates that no system may strictly satisfy the coverage requirement if the number of satellites in a system is fewer than the theoretical lower bound. Therefore, this study utilizes the percent coverage as an appropriate figure of merit for quantifying the coverage performance of satellite constellations relative to the desired coverage performance. The percent coverage computation from the coverage timeline $\boldsymbol{b}_\text{B}$ and the desired coverage $\boldsymbol{r}_\text{B}$ is explained. If the coverage timeline is greater than or equal to the desired coverage at discrete-time instant n, then we consider the coverage is satisfied at that discrete-time instant ($\delta[n] \in \{0, 1\}$ being the indicator). Hence, the percent coverage is computed by summing up all coverage satisfactoriness indicators and dividing it by the cardinality of the discrete-time set, which is L. The goal is to maximize the percent coverage objective function. Under the minimization problem, the objective function is:

$$J_1 = -\frac{100}{L}\sum_{n=0}^{L-1}\delta[n] \tag{10}$$

where $\delta[n]$ is defined as:

$$\delta[n] = \begin{cases} 1, & \text{if } b[n] \geq r[n] \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

Note that Eq. (11) involves a nonlinear conditional statement. This can be linearized by formulating it via the Big $\mathcal{M}$ method and introducing binary variables. Let $u[n] \in \{0, 1\}$ be a binary variable that is true (1, in boolean algebra) when $b[n] \geq r[n]$ and that is false (0) otherwise.

$$b[n] \geq r[n] \Leftrightarrow u[n] = 1$$

This equivalence can be represented in the following way by employing the Big $\mathcal{M}$ method, where $\mathcal{M}$ is a sufficiently large number:

$$b[n] \geq r[n] - \mathcal{M}(1 - u[n]) \tag{12a}$$

$$b[n] \leq r[n] + \mathcal{M}u[n] \tag{12b}$$

To illustrate how this method works, knowing the fact that {if $b[n] \geq r[n]$, then $u[n] = 1$}, then, Eq. (12a) becomes $b[n] \geq r[n]$ and Eq. (12b) becomes $b[n] \leq r[n] + \mathcal{M}$. In order for these two inequalities to be true, $\mathcal{M}$ must be a sufficiently large number, of which this article utilizes $\mathcal{M} = 10^3$.

Now that the conditional statement is linearized as shown in Eqs. (12), we have to couple each condition with its corresponding value. Define another variable $\delta[n]$ that varies based on the condition of $u[n]$:

$$u[n] = 1 \Rightarrow \delta[n] = 1 \tag{13a}$$

$$u[n] = 0 \Rightarrow \delta[n] = 0 \tag{13b}$$

Eqs. (13) yield a total of four inequality constraints for each n:

$$1 - \mathcal{M}(1 - u[n]) \leq \delta[n] \leq 1 + \mathcal{M}(1 - u[n]) \tag{14a}$$
$$-\mathcal{M}u[n] \leq \delta[n] \leq \mathcal{M}u[n] \tag{14b}$$

Combining, Eqs. (12) and Eqs. (14), there are six inequality constraints for each discrete-time instant n. The conditional statements are successfully linearized. These inequality constraints are referred to as the Big $\mathcal{M}$ method constraints.

An important observation is made—a one-to-one same-value matching between $u[n]$ and $\delta[n]$ in Eq. (13). Therefore, we will simply ignore constraints in Eq. (14) hereafter to reduce the total number of constraints per discrete-time instant n from six to two. This effort also halves the total number of design variables by not considering $\boldsymbol{\delta}$ variables (without it, there are $\boldsymbol{u}$ and $\boldsymbol{\delta}$ variables). Hence, the objective function J_1 in Eq. (10) can be rewritten in terms of $u[n]$ instead of $\delta[n]$:

$$J_1 = -\frac{100}{L}\sum_{n=0}^{L-1} u[n] \tag{15}$$

Cost All reconfiguration processes incur costs except for the case of an isomorphic reconfiguration process, in which all configuration constituent design variables and satellite assignments remain unchanged during the process. In this paper, we consider the sum of the total ΔV's consumed by all system satellites as the cost, that is:

$$J_2 = \text{cost}(\varphi_{\text{AB}}) = \sum_{k=0}^{N} \Delta V_k = \sum_{n=0}^{L-1} c[n]f[n] \tag{16}$$

We assume that the ΔV consumption is the only cost factor and the time to reconfigure does not affect the performance of the system; the system this paper is dealing with is not conducting a reconfiguration process in response to time-critical mission requirements. However, note that the time cost is a reasonable cost factor to consider in practice.

Weighted Sum Objective Function To consider both the coverage figure of merit and the sum of ΔV's used by all satellites, the weighted sum approach is used to define a new objective function J:

$$\begin{aligned} J &\triangleq w_1 J_1 + w_2 J_2 \\ &= -w_1 \frac{100}{L}\sum_{n=0}^{L-1} u[n] + w_2 \sum_{n=0}^{L-1} c[n]f[n] \end{aligned} \tag{17}$$

where w_1 and w_2 are the weighting factors for J_1 and J_2, respectively.

Note that the units for J_1 and J_2 do not match. The cost estimation in dollars for J_2 is trivial; the cost conversion is in the form of $ per km/s. However, the cost conversion for J_1 is nontrivial, which requires in the form of $ per coverage percentage; such a metric requires the modeling of a loss cost for not meeting one or more satellites in view. Hence, the linear combination of these two objective functions dictates the unit to be dimensionless. The normalization of two objective functions is considered as part of the weighting factors.

Coupling the Constellation Design and Transfer Problems

This subsection examines the nature of the constellation design problem and the constellation transfer problem and attempts to couple them together. The following Eqs. (18) and (19) lists nominal optimization formulations for transfer and design problem, respectively.

$$\begin{aligned} \min_{\boldsymbol{f}} \quad & \sum_{k=1}^{N}\sum_{n=0}^{L-1} C[k,n]f[k,n] \\ \text{s.t.} \quad & \sum_{k=1}^{N} f[k,n] = 1, \;\; n = 0, ..., L-1 \\ & \sum_{n=0}^{L-1} f[k,n] \leq 1, \;\; k = 1, ..., N \\ & f[k,n] \in \{0,1\}, \;\; \forall k,n \end{aligned} \tag{18}$$

Unbalanced Linear Assignment Problem

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \sum_{n=0}^{L-1} x[n] \\ \text{s.t.} \quad & \sum_{l=0}^{L-1} V[n,l]x[l] \geq r[n], \;\; n = 0, ..., L-1 \\ & x[n] \in \{0,1\}, \;\; \forall n \end{aligned} \tag{19}$$

Regional Constellation Design Problem[9]

The constellation transfer problem can be modeled as a linear assignment problem (LAP). The transportation problem in general mandates that its design variables to be flows $\boldsymbol{f}$ (i.e., edges) between the nodes of given bipartite sets. In this context, the sets here refer to a set of existing satellites for configuration A (N nodes) and a set of orbital slots for configuration B (L nodes). This leads to the combinatorial nature of the problem in which every node in a set A spans L edges that directs to every node in a set B. Since the number of nodes in set B is larger than that of set A (i.e., $L > N$), we form an unbalanced assignment problem. Hence, the problem is in a binary integer linear programming form as shown in Eq. (18). The cost of each flow is represented as a matrix $\boldsymbol{C}$. In this paper, the cost matrix is hard-coded based on the strategy outlined in Reference 19; the orbital transfer strategy assumes a simultaneous inclination/RAAN change and the super-synchronous phasing for mean anomaly correction.

On the other hand, the optimization of a regional satellite constellation configuration design following the APC decomposition requires the design variables to be constellation pattern vector $\boldsymbol{x}$, which represents discrete-time shifts that indicate the relative temporal positions of satellites with respect to the seed satellite. The regional constellation design problem based on the APC decomposition is shown in Eq. (19), which is also in the form of binary integer linear programming.

Both LAP and regional constellation design problems are in the form of BILP. However, each problem is formulated with a different type of design variables (i.e., flows vs. constellation pattern vector) and a different number of design variables (NL vs. N).

The assignment problem and the design problem based on the APC decomposition method can be coupled as follows:

$$\boxed{x[n] = \sum_{k=1}^{N} f[k,n], \quad n = 0, ..., L-1} \tag{20}$$

This coupled relationship enables an integrated BILP formulation that simultaneously considers both the constellation transportation problem and the constellation design problem; both the regional coverage and the N-fixed formulation aspects are embedded in this relationship. This integration is the main contribution of this paper. The following section introduces the integrated BILP problem.

PROBLEM FORMULATION

This section introduces the general problem statement and the integrated BILP formulation to solve a satellite constellation reconfiguration problem that incorporates both the regional coverage and N-fixed formulation aspects. An optimization formulation is introduced such that it can serve as the baseline formulation for optimizations with special cases. Further, several additional features to the baseline formulation are discussed such that system operators can adapt the framework depending on the use cases.

General Problem Statement

Suppose a common RGT satellite constellation system undertakes a reconfiguration process $\boldsymbol{\varphi}_{\text{AB}}$ from its initial configuration $\mathcal{G}_{\text{A}}$ to a final configuration $\mathcal{G}_{\text{B}}$, of which is unknown. The reconfiguration process is subject to some combination of variations in mission operations. The problem is to find the specifications of φ_{AB} (i.e., both design and assignment) that maximizes the utility.

Baseline Optimization Formulation

The baseline optimization formulation is of the form Binary Integer Linear Program (BILP):

$$\boxed{\begin{array}{lll}
\underset{\boldsymbol{d}}{\text{minimize}} & J = -w_1 \dfrac{100}{|\mathcal{J}|\,L} \displaystyle\sum_{n=0}^{|\mathcal{J}|L-1} u[n] + w_2 \sum_{n=0}^{|\mathcal{Z}|NL-1} c[n] f[n] & \\
\text{subject to} & \boldsymbol{h}_1 = \displaystyle\sum_{n=0}^{|\mathcal{Z}|NL-1} f_k[n] = 1, & k = 1, ..., N \\
& \boldsymbol{g}_1 = \displaystyle\sum_{k=1}^{N} f_k[n] \leq 1, & n = 0, ..., |\mathcal{Z}|\,NL-1 \\
& \boldsymbol{g}_2 = -\displaystyle\sum_{l=0}^{|\mathcal{Z}|NL-1} \Big(V[n,l]x[l]\Big) + \mathcal{M}u[n] \leq -r[n] + \mathcal{M}, & n = 0, ..., |\mathcal{Z}|\,NL-1 \\
& \boldsymbol{g}_3 = \displaystyle\sum_{l=0}^{|\mathcal{Z}|NL-1} \Big(V[n,l]x[l]\Big) - \mathcal{M}u[n] \leq r[n], & n = 0, ..., |\mathcal{Z}|\,NL-1 \\
& \boldsymbol{d} \in \mathbb{Z}_2^{(|\mathcal{Z}|NL+|\mathcal{J}|L)} &
\end{array}} \tag{21}$$

where we have the following definitions of the design variable vector $\boldsymbol{d}$:

$$\boldsymbol{f} = [\boldsymbol{f}_1, ..., \boldsymbol{f}_N]^T \tag{22a}$$

$$\boldsymbol{u} = [\boldsymbol{u}_1, ..., \boldsymbol{u}_{|\mathcal{J}|}]^T \tag{22b}$$

$$\boldsymbol{d} \triangleq [\boldsymbol{f}, \boldsymbol{u}]^T \tag{22c}$$

The flow variables $\boldsymbol{f}$ follow the notation convention as shown in Figure 3. The artificial variables $\boldsymbol{u}$ are added due to the Big $\mathcal{M}$ method. There are $|\mathcal{Z}|\,NL$ number of $\boldsymbol{f}$ variables and $|\mathcal{J}|\,L$ number of $\boldsymbol{u}$ variables. Hence, there are $|\mathcal{Z}|\,NL + |\mathcal{J}|\,L$ design variables.

The constraints $\boldsymbol{h}_1$ and $\boldsymbol{g}_1$ are the assignment problem-related constraints; $\boldsymbol{h}_1$ guarantee that there should be only one out-going flow (i.e., matching) per satellite and $\boldsymbol{g}_1$ guarantee that each orbital slot in B may host at most one satellite (i.e., unbalanced linear constraint). The constraints $\boldsymbol{g}_2$ and $\boldsymbol{g}_3$ are the Big $\mathcal{M}$ method constraints as introduced previously.

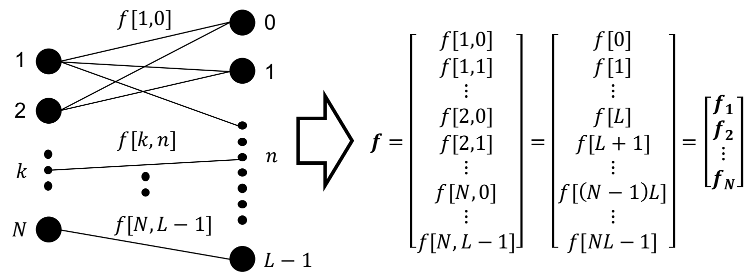


Figure 3: Design variable convention (assuming $|\mathcal{Z}| = 1$)

Additional Features to the Baseline Formulation

The following additional features are handy depending on the situation in addition to the baseline optimization formulation as shown in Eq. (21).

To enforce the maximum ΔV consumption constraint on each satellite, the $\boldsymbol{g}_4$ constraints can be additionally used:

$$\begin{aligned} \boldsymbol{g}_4 &\triangleq \Delta V_k \leq \Delta V_{\max,k}, \quad k = 1, ..., N \\ &= c[n]f[n] \leq \Delta V_{\max,k}, \quad n = 0, ..., |\mathcal{Z}|\, NL - 1 \end{aligned} \tag{23}$$

With these constraints, it is logical to replace the objective function from J to J_1 such that the optimization problem becomes the coverage percentage maximization problem given the individual maximum ΔV constraints:

$$\min_{\boldsymbol{d}} J_1 \ \text{ s.t. } \{\boldsymbol{h}_1, \boldsymbol{g}_1, \boldsymbol{g}_2, \boldsymbol{g}_3, \boldsymbol{g}_4, \boldsymbol{d} \in \mathbb{Z}_2^{(|\mathcal{Z}|NL + |\mathcal{J}|L)}\} \tag{24}$$

To enforce the maximum total ΔV consumption constraint used by all system satellites, the following g_5 constraint can be additionally used:

$$\begin{aligned} g_5 &\triangleq \sum_{k=1}^{N} \Delta V_k \leq \Delta V_{\max} \\ &= \sum_{n=0}^{|\mathcal{Z}|NL-1} c[n]f[n] \leq \Delta V_{\max} \end{aligned} \tag{25}$$

Similarly, it is logical to formulate the coverage percentage maximization problem:

$$\min_{\boldsymbol{d}} J_1 \ \text{ s.t. } \{\boldsymbol{h}_1, \boldsymbol{g}_1, \boldsymbol{g}_2, \boldsymbol{g}_3, g_5, \boldsymbol{d} \in \mathbb{Z}_2^{(|\mathcal{Z}|NL + |\mathcal{J}|L)}\} \tag{26}$$

Packing and Unpacking Procedures

In this section, we introduce two procedures to properly code and decode the variables and parameters. The packing procedure is critical to ensure proper implementation of the program. This is shown in Algorithm 1. Note that, there exists an inherent weighting factor scheme for each circulant matrix in Eq. (27) (the default is the equal weight). Once the optimization returns an optimal solution $\boldsymbol{d}^*$, the unpacking procedure is required to decode the result as shown in Algorithm 2.

Algorithm 1 Packing $\boldsymbol{V}$ and $\boldsymbol{r}$

procedure PACK(**œ**$^{(1)}$, ..., **œ**$^{(|\mathcal{Z}|)}$, $\boldsymbol{r}_1, ...\boldsymbol{r}_{|\mathcal{J}|}$)

Define an augmented circulant matrix $\bar{\boldsymbol{V}}$:

$$\bar{\boldsymbol{V}} \triangleq \begin{bmatrix} \boldsymbol{V}_1^{(1)} & \boldsymbol{V}_1^{(2)} & \cdots & \boldsymbol{V}_1^{(|\mathcal{Z}|)} \\ \boldsymbol{V}_2^{(1)} & \boldsymbol{V}_2^{(2)} & \cdots & \boldsymbol{V}_2^{(|\mathcal{Z}|)} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{V}_{|\mathcal{J}|}^{(1)} & \boldsymbol{V}_{|\mathcal{J}|}^{(2)} & \cdots & \boldsymbol{V}_{|\mathcal{J}|}^{(|\mathcal{Z}|)} \end{bmatrix}_{(|\mathcal{J}|L \times |\mathcal{Z}|L)} \tag{27}$$

Repetitively extend the augmented circulant matrix $\bar{\boldsymbol{V}}$ by N times to get the new circulant matrix $\boldsymbol{V}$:

$$\boldsymbol{V} \triangleq \begin{bmatrix} \bar{\boldsymbol{V}} & \cdots & \bar{\boldsymbol{V}} \end{bmatrix}_{(|\mathcal{J}|L \times |\mathcal{Z}|NL)}$$

Define an augmented coverage requirements vector $\boldsymbol{r}$:

$$\boldsymbol{r} \triangleq [\boldsymbol{r}_1, ..., \boldsymbol{r}_{|\mathcal{J}|}]^T_{(|\mathcal{J}|L \times 1)}$$

Return $\boldsymbol{V}$ and $\boldsymbol{r}$

end procedure

Algorithm 2 Unpacking $\boldsymbol{d}^*$

procedure UNPACK($\boldsymbol{d}^*$)

The argument of the minimum $\boldsymbol{d}^*$ is in the form of:

$$\boldsymbol{d}^* = [\boldsymbol{f}^*, \boldsymbol{u}^*]^T$$

Decompose the optimal design variable $\boldsymbol{f}^* \in \mathbb{Z}_2^{(|\mathcal{Z}|NL)}$ into a set of N equal-length segments:

$$\boldsymbol{f}^* \mapsto [\boldsymbol{f}_1^*, ..., \boldsymbol{f}_N^*]^T$$

Do element-wise sum over all segments $\boldsymbol{f}_1^*, ..., \boldsymbol{f}_N^*$ to create an aggregated vector $\bar{\boldsymbol{x}}_{\mathrm{B}}^*$:

$$\bar{x}_{\mathrm{B}}^*[n] \triangleq \sum_{k=0}^{N} f_k^*[n], \quad n = 0, ..., |\mathcal{Z}|\, L - 1$$

If a system consists of $|\mathcal{Z}|$ subconstellations, decompose the augmented vector $\bar{\boldsymbol{x}}_{\mathrm{B}} \in \mathbb{Z}_2^{(|\mathcal{Z}|L)}$ into $|\mathcal{Z}|$ segments:

$$\bar{\boldsymbol{x}}^* \mapsto [\boldsymbol{x}_{\mathrm{B}}^{(1)*}, ..., \boldsymbol{x}_{\mathrm{B}}^{(|\mathcal{Z}|)*}]^T$$

Return $\boldsymbol{x}_{\mathrm{B}}^{(1)*}, ..., \boldsymbol{x}_{\mathrm{B}}^{(|\mathcal{Z}|)*}$

end procedure

Optimization Problem Formulation for Strict Coverage Requirement

This subsection explores the case when the optimization now necessitates the coverage requirement strictness. That is, the configuration B must strictly fulfill the new coverage requirement $\boldsymbol{r}_{\mathrm{B}}$. There are two ways to formulate this problem: either via setting all $\boldsymbol{u}$ variables to one or adding strict coverage requirement constraints. Implementing the latter, one can remove all Big $\mathcal{M}$-related variables and constraints (i.e., no $\boldsymbol{u}$ variables nor $\boldsymbol{g}_2$ and $\boldsymbol{g}_3$ constraints). Formulating the optimization problem:

$$
\begin{aligned}
\underset{\boldsymbol{f}}{\text{minimize}} \quad & J_2 = \sum_{n=0}^{|\mathcal{Z}|NL-1} c[n]f[n] \\
\text{subject to} \quad & \boldsymbol{h}_1 = \sum_{n=0}^{|\mathcal{Z}|L-1} f_k[n] = 1, \quad k = 1, ..., N \\
& \boldsymbol{g}_1 = \sum_{k=1}^{N} f_k[n] \leq 1, \quad n = 0, ..., |\mathcal{Z}|\, L-1 \\
& \boldsymbol{g}_6 = \boldsymbol{V}\boldsymbol{x} \geq \boldsymbol{r} \\
& \boldsymbol{f} \in \mathbb{Z}_2^{(|\mathcal{Z}|NL)}
\end{aligned}
\tag{28}
$$

Unlike the coverage percentage maximization problems in Eq. (24) and Eq. (26), the optimization in Eq. (28) is a minimization problem with respect to the cost function. This is since the coverage requirement is strictly satisfied via constraints $\boldsymbol{g}_6$. A problem can be infeasible if it cannot satisfy the coverage requirement with the given number of satellites (i.e., less than the minimum number of satellites required to strictly fulfill the coverage requirement). However, this can be avoided by presolving for the minimum number of satellites required via the original APC decomposition method as shown in Eq. (19). The difference is the number of newly launched satellites.

ILLUSTRATIVE EXAMPLE

This section demonstrates the general applicability of the reconfiguration framework developed in this paper.

Synopsis

Suppose a satellite constellation system with five satellites spanning in two-subconstellations undertakes a reconfiguration process due to variations in mission operations. The system operators wish to reconfigure the existing constellation system such that it provides increased coverage over new areas that are recently affected by natural disasters: Getty, California and Asheikri, Nigeria. The new configuration is aided with α number of new satellites.

Nominal Configuration

The five-satellite two-subconstellation system has the following initial configuration state, $\mathcal{G}_{\mathrm{A}} = \{\boldsymbol{œ}_{0,\mathrm{A}}^{(1)}, \boldsymbol{œ}_{0,\mathrm{A}}^{(2)}, \boldsymbol{x}_{\mathrm{A}}^{(1)}, \boldsymbol{x}_{\mathrm{A}}^{(2)}\}$. The seed satellite orbital elements vectors are:

$$
\begin{aligned}
\boldsymbol{œ}_{0,\mathrm{A}}^{(1)} &= [10\,527.4\,\mathrm{km}, 0, 70°, 0°, 0°, 0°]^T \\
\boldsymbol{œ}_{0,\mathrm{A}}^{(2)} &= [12\,758.4\,\mathrm{km}, 0, 47.92°, 0°, 0°, 0°]^T
\end{aligned}
$$

The constellation pattern vectors are:

$$x_{\mathrm{A}}^{(1)}[n] = \begin{cases} 1, & \text{for } n = 67, 155, 285 \\ 0, & \text{otherwise} \end{cases}$$

$$x_{\mathrm{A}}^{(2)}[n] = \begin{cases} 1, & \text{for } n = 199, 399 \\ 0, & \text{otherwise} \end{cases}$$

Interpreting, there are three satellites placed in $4149.2\,\mathrm{km}$-altitude circular orbits and two satellites placed in $6380.2\,\mathrm{km}$-altitude circular orbits. Figure 6a illustrates the system in the Earth-centered inertial (ECI) frame.

Variations in Mission Operations

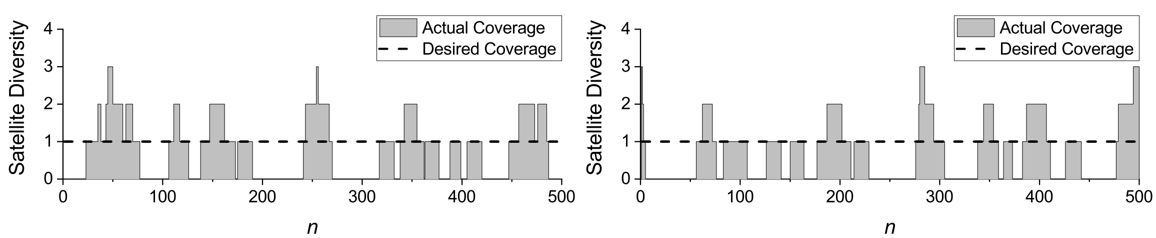
Change in the area of interest is demanded. The new area of interest $\mathcal{J}_{\mathrm{B}}$ consists of two target points, Getty, California and Asheikri, Nigeria (shown in Figure 6a):

$$\mathcal{J}_{\mathrm{B}} = \Big\{ \big(\phi = 34.09°\mathrm{N}, \lambda = 118.47°\mathrm{W}\big), \big(\phi = 12.93°\mathrm{N}, \lambda = 11.96°\mathrm{E}\big) \Big\}$$

We wish to maximize the coverage over both target points of interest (both targets have equal weights). Hence, the desired coverage vectors for all target points can be defined as:

$$\boldsymbol{r}_{1,\mathrm{B}} = \boldsymbol{r}_{2,\mathrm{B}} = \mathbf{1}$$

Assuming $\varepsilon_{\mathrm{min},1} = \varepsilon_{\mathrm{min},2} = 10°$, the initial configuration $\mathcal{G}_{\mathrm{A}}$ provides 54.4% and 50.6% coverage over $j = 1$ and $j = 2$, respectively. The corresponding coverage timelines over the new areas of interest with the initial configuration $\mathcal{G}_{\mathrm{A}}$ are shown in Figure 4. As one can observe, the initial configuration provides poor coverage performances on both targets.



(a) Coverage timeline over Getty, California ($\boldsymbol{b}_1$) (b) Coverage timeline over Asheikri, Nigeria ($\boldsymbol{b}_2$)

Figure 4: Coverage timelines with the initial configuration $\mathcal{G}_{\mathrm{A}}$

Additionally, we assume a batch launch of two satellites for the new configuration, $\alpha = 2$. These two new satellites are deployed simultaneously together from a launcher at a position in space $x_{\mathrm{A}}^{(1)}[0]$ (same as $\boldsymbol{œ}_{0,\mathrm{A}}^{(1)}$). Therefore, the total number of satellites for the final configuration is $N_{\mathrm{B}} = N_{\mathrm{A}} + \alpha = 7$.

Lastly, without loss of generality, the seed satellite orbital elements vectors of the new configuration are assumed to remain the same (note that this does not mean that satellites must remain within their own subconstellation, but they are free to relocate per optimization):

$$\boldsymbol{œ}_{0,\mathrm{A}}^{(1)} = \boldsymbol{œ}_{0,\mathrm{B}}^{(1)} \quad \text{and} \quad \boldsymbol{œ}_{0,\mathrm{A}}^{(2)} = \boldsymbol{œ}_{0,\mathrm{B}}^{(2)}$$

Hence, the goal of the optimization is to find the distribution of satellites, i.e., $\boldsymbol{x}_{\mathrm{B}}^{(1)}$ and $\boldsymbol{x}_{\mathrm{B}}^{(2)}$, that maximizes the coverage over the new target points of interest while minimizing the reconfiguration ΔV costs.

Results

The formulation of the optimization problem is identical to that of the baseline in Eq. (21). The weights for each objective function is set to $w_1 = 1\,/\%$ and $w_2 = 1\,/\mathrm{km/s}^*$. In addition to this baseline case, we additionally explore different combinations of constraints and objective functions to better understand the trade space.

The argument of the minimization is obtained:

$$\boldsymbol{d}^* = \arg\min_{\boldsymbol{d}} J \;\text{ s.t. } \{\boldsymbol{h}_1, \boldsymbol{g}_1, \boldsymbol{g}_2, \boldsymbol{g}_3, \boldsymbol{d} \in \mathbb{Z}_2^{(|\mathcal{Z}|NL+|\mathcal{J}|L)}\}$$

where $\boldsymbol{x}_{\mathrm{B}}^{(1)}$ and $\boldsymbol{x}_{\mathrm{B}}^{(2)}$ can be deduced from $\boldsymbol{d}^*$ following the unpacking procedure (Algorithm 2):

$$x_{\mathrm{B}}^{(1)}[n] = \begin{cases} 1, & \text{for } n = 285 \\ 0, & \text{otherwise} \end{cases}$$

$$x_{\mathrm{B}}^{(2)}[n] = \begin{cases} 1, & \text{for } n = 27, 78, 144, 199, 406, 459 \\ 0, & \text{otherwise} \end{cases}$$

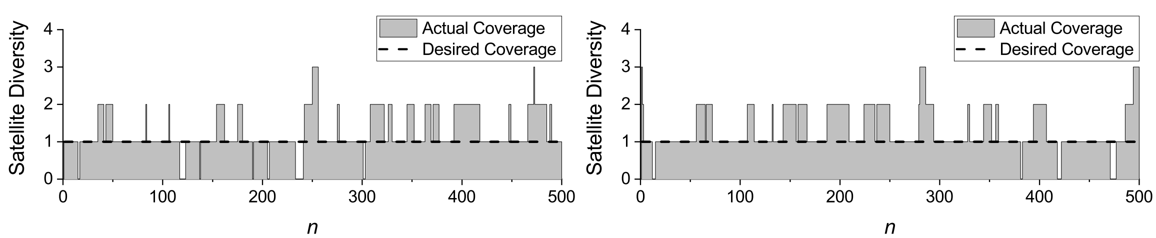
The corresponding objective function value is:

$$J^*(\boldsymbol{d}^*) = -81.6$$

Decomposing J^*,

$$J_1^* = -96.3 \quad \text{and} \quad J_2^* = 14.7$$

where J_1 is the mean value of two percent coverage metrics over two target points: the configuration $\mathcal{G}_{\mathrm{B}}$ provides $95.6\,\%$ and $97\,\%$ coverage over $j = 1$ and $j = 2$ targets, respectively (see Figure 5). This is the significant improvement in the coverage over both target points of interest. J_2 can be translated as the total ΔV consumed by all system satellites including the newly launched ones, hence, $14.7\,\mathrm{km/s}$.



(a) Coverage timeline over Getty, California ($\boldsymbol{b}_1$) (b) Coverage timeline over Asheikri, Nigeria ($\boldsymbol{b}_2$)

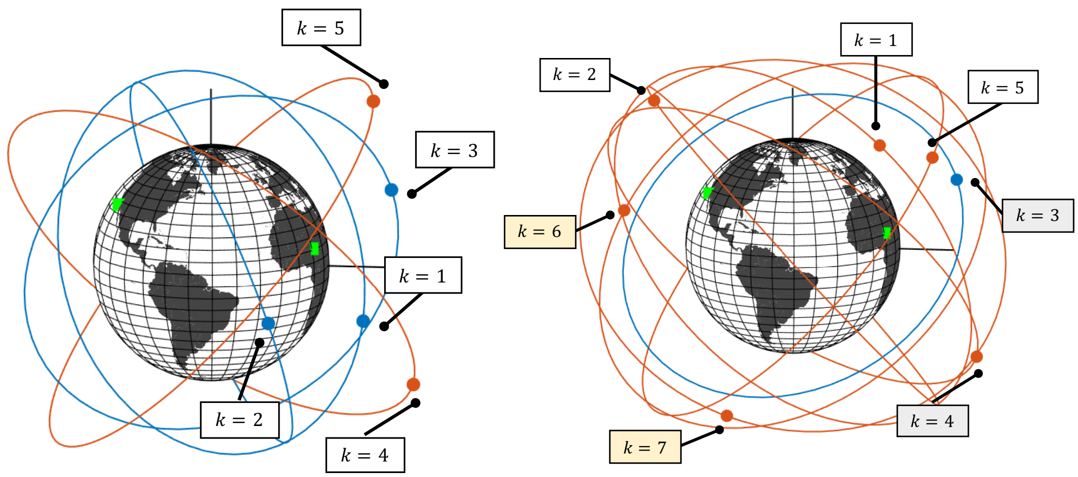
Figure 5: Coverage timelines with the optimized configuration $\mathcal{G}_{\mathrm{B}}$

*The units of the weighting factors are chosen such that the weighted-sum objective function is non-dimensionalized.

The labeling of satellites and the observation of where each satellite is relocated are provided in Table 1. The satellite indices 6 and 7 refer to those of the newly added satellites. Out of seven satellites, five satellites ($k = 1, 2, 5, 6, 7$) are relocated into new orbital slots. Only $k = 5$ satellite moved within the same subconstellation regime, but the rest of the four satellites moved from the subconstellation 1 regime to the subconstellation 2 regime. The mean ΔV used by the system satellites is $2.94\,\mathrm{km/s}$. Figure 6b shows the final configuration in the ECI frame.

Table 1: Constellation pattern vector analysis

Satellite index k	Position in configuration A	Position in configuration B	ΔV (km/s)
1	$x_A^{(1)}[67]$	$x_B^{(2)}[78]$	3.59
2	$x_A^{(1)}[155]$	$x_B^{(2)}[144]$	2.90
3	$x_A^{(1)}[285]$	$x_B^{(1)}[285]$	0
4	$x_A^{(2)}[199]$	$x_B^{(2)}[199]$	0
5	$x_A^{(2)}[399]$	$x_B^{(2)}[406]$	0.53
6 (newly added)	$x_A^{(1)}[0]$	$x_B^{(2)}[27]$	3.44
7 (newly added)	$x_A^{(1)}[0]$	$x_B^{(2)}[459]$	4.24



(a) Initial configuration $\mathcal{G}_A$ (b) Final configuration $\mathcal{G}_B$

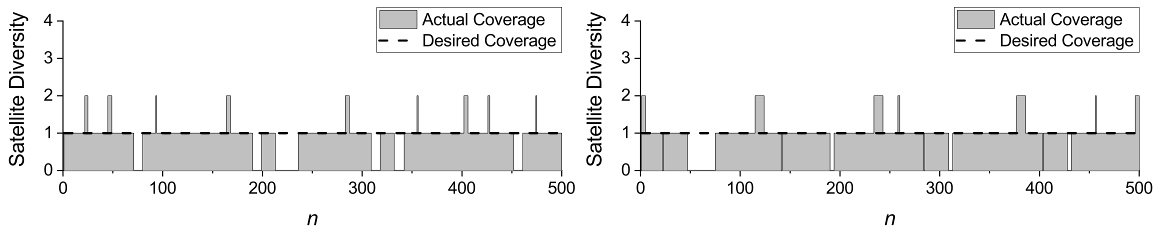
Figure 6: Side-by-side comparison of the initial configuration and the final configuration in the ECI frame. The inner blue orbits refer to the subconstellation $z = 1$ and the outer orange orbits refer to the subconstellation $z = 2$. Note that $k = 3, 4$ satellites remain unchanged.

One can solve for the same optimization problem with just the coverage percentage metric objective function J_1 (no cost factor is considered) to identify the maximum percentage it can provide with $N_B = 7$ satellites. In this case, the result yields $J_1 = -98.4$, which corresponds to $96.8\,\%$ over target point $j = 1$ and $100\,\%$ over target point $j = 2$.

Additional Considerations

Zero Satellite Addition Considering $\alpha = 0$ and with the baseline optimization, one can get $J = -76.29$ ($J_1 = -88.7$ and $J_2 = 12.41$). The percent coverage for each target point of interest is:

$86.2\,\%$ for $j = 1$ and $91.2\,\%$ for $j = 2$ target, respectively (see Figure 7). This is a significant improvement in the percent coverage compared to the initial configuration: 54.4% and 50.6% for $j = 1$ and $j = 2$, respectively. Hence, even without the addition of satellites, this optimization demonstrates the value of the satellite constellation reconfiguration and the framework.



(a) Coverage timeline over Getty, California ($\boldsymbol{b}_1$)   (b) Coverage timeline over Asheikri, Nigeria ($\boldsymbol{b}_2$)

Figure 7: Coverage timelines (no satellites added)

Individual ΔV *Constraints* Assuming $\Delta V_{\text{max},k} = \Delta V_{\text{max}}, \ \forall k$ and solving the optimization problem shown in Eq. (24) while keeping the rest of parameters the same, one can get the following results as shown in Table 2.

Table 2: Individual ΔV Constraints

ΔV_{max} (km/s)	J_1^*	% cov. $j = 1$	% cov. $j = 2$
1	-63.3	$68.0\,\%$	$58.6\,\%$
2	-75.0	$76.4\,\%$	$73.6\,\%$
3	-95.0	$93.6\,\%$	$96.4\,\%$
4	-98.4	$96.8\,\%$	$100\,\%$

The result of the $\Delta V_{\text{max}} = 3\,\text{km/s}$ case outperforms the zero satellite addition case, this is because the objective function is solely the coverage maximization. The $\Delta V_{\text{max}} = 4\,\text{km/s}$ case is essentially identical to that of without any ΔV restriction.

ΔV *Sum Constraint* Instead of setting the maximum ΔV values for individual satellites, we can set up ΔV_{max} on the sum of ΔV's used by all satellites. Solving the optimization problem in Eq. (26) while keeping the rest of the parameters the same, we get the following results as shown in Table 3.

Table 3: ΔV Sum Constraint

ΔV_{max} Sum (km/s)	J_1^*	% cov. $j = 1$	% cov. $j = 2$
1	-58.8	$61.4\,\%$	$56.2\,\%$
5	-72.0	$73.0\,\%$	$71.0\,\%$
10	-87.0	$82.2\,\%$	$91.8\,\%$
15	-96.5	$96.2\,\%$	$96.8\,\%$

One can observe that even with ΔV_{max} sum of $1\,\text{km/s}$, the reconfigured system outperforms the initial configuration. Increase in ΔV_{max} starts to mimic the result obtained without setting any ΔV constraint.

Strict Coverage Requirement Solving the optimization problem shown in Eq. (28), one would need at least 8 satellites to strictly satisfy single-fold continuous coverage over both $j = 1$ and $j = 2$ target points concurrently. This indicates $\alpha = 3$. One can achieve this with $J_2 = 23.42$ (i.e., the sum of ΔV's is equal to $23.42\,\mathrm{km/s}$). A total of seven satellites require relocation.

CONCLUSION

This paper presents a method for reconfiguration of satellite constellations under various combinations of mission operational variations. The main intellectual merit delivered by this paper is in the integration of constellation design and constellation transfer problems that are otherwise serial and independent in nature. The integrated optimization problem is in the form of a modified linear assignment problem (i.e., a binary integer linear programming) and enables the use of global optimization algorithms to find globally-optimal solution(s). Two core design philosophies—N-fixed formulation and regional coverage—that justify and reinforce the utility of satellite constellation reconfiguration are successfully incorporated into the formulation. The method is the extension of the N-flexible regional coverage constellation design method, in which a combination of both provides a set of streamlined design and operational analysis tools for system designers and operators. The case study attests to the value of the proposed framework by demonstrating its efficacy in various reconfiguration settings.

Several promising future works are discussed. The optimization problem is in the form of LAP plus additional constraints (e.g., the Big $\mathcal{M}$ method), thereby making the problem NP-hard if it were to be solved by nominal mixed-integer linear programming solvers (e.g., Gurobi). One may modify an established algorithm such as the Hungarian algorithm or the auction algorithm to guarantee the polynomial problem runtime. Furthermore, because of its form, one can formulate a network flow optimization problem such that it quantifies an economic critical point that manifests "when reconfiguring makes sense" versus "when launching a whole new constellation makes sense" given a new set of mission requirements. Any mixed strategy (i.e., launch some and reconfigure some) is only contingent upon the construction of a complete design space that incorporates both the constellation design problem and the constellation transfer problem. This will provide important business strategic insights for system stakeholders in addition to the tools provided for system designers and operators.

ACKNOWLEDGMENTS

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NOTATION

a	semi-major axis
$\boldsymbol{b}$	coverage timeline
c	cost vector
C	cost matrix
$\boldsymbol{d}$	design vector
e	eccentricity
$\boldsymbol{f}$	flow variables
$\mathcal{G}$	satellite constellation configuration set
i	inclination
$\mathcal{J}$	set of target points
L	length (number of time steps) of vectors
M	mean anomaly
n	discrete-time instant
N	number of satellites
$\boldsymbol{œ}$	orbital elements vector
$\boldsymbol{r}$	desired coverage vector
$\boldsymbol{v}$	access profile
V	access profile circulant matrix
$\boldsymbol{x}$	constellation pattern vector
$\mathcal{Z}$	set of subconstellations
α	change in number of satellites
ω	argument of perigee
Ω	right ascension of ascending node
ε	elevation angle

Subscripts

j	target index

Superscripts

z	subconstellation index

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